On the Achievable Average Power Reduction of MSM Optical Signals *

MASOUD SHARIF and BABAK HASSIBI
Department of Electrical Engineering
California Institute of Technology, Pasadena, CA 91125
Email: {masoud,hassibi}@systems.caltech.edu
Fax Number: (626) 564 9307
Phone Number: (626) 395 2215

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Abstract

In this letter, we consider the achievable average power reduction of multiple subcarrier modulated optical signals by using optimized reserved carriers. Based on Nehari's result we present a lower bound for the maximum average power of the signal after adding the reserved carriers. Simulations show that the mean value of the average required power behaves very close to $\sqrt{2n\log\log n}$ for BPSK constellations where n is the number of subcarriers. We further remark on evaluating optimum values for reserved carriers using convex optimization and Nehari's result.

Key Words–optical communication, multiple subcarrier modulation, intensity modulation, average power reduction, Nehari's theorem.

Corresponding Author– Masoud Sharif

1 Introduction

Multiple subcarrier intensity modulation (IM) with direct detection (DD) is an attractive technique for high speed wireless optical communications [1, 2, 3]. In multiple subcarrier modulation (MSM) with IM/DD, the transmit signal consists of several subcarriers and is modulated onto the optical carrier by intensity modulation. The main disadvantage of MSM with IM/DD is its low optical average power efficiency which is essentially the same problem as the high peak to mean envelope power ratio (PMEPR) of multicarrier signals in RF communication systems. Since the optical intensity should be always positive, the signal should have a dc component to guarantee

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that the minimum of the signal is greater than zero. Since this dc component is proportional to the average optical power, as the number of carriers n, increases and the signal exhibits smaller minimum values, the required average power of the system will be increased.

Previously, the clipping method has been considered to increase the average power efficiency [3]. Recently, You and Kahn [2] proposed block coding and tone reservation to maximize the minimum value of the multiple subcarrier signal. The tone reservation is a method in which the dummy carriers with optimized amplitude are used to increase the average power efficiency of the system. It is worth noting that the tone reservation problem here is a bit different from that of OFDM (Orthogonal Frequency Division Multiplexing) in the sense that we are just interested in the maximization of the minimum value of the signal unlike OFDM in which we are interested in the ratio of the maximum of the signal to the average power [4].

In this letter, we address the achievable average power reduction by using optimum reserved tones appended at the end of the signal. Using Nehari's result on the estimation of a causal function by an anti-causal one, we propose an achievable lower bound on the maximum average power of MSM signals by using reserved tones. We further remark on how to evaluate the optimum values for the reserved carriers using convex optimization and Nehari's result.

2 Multiple Subcarrier Modulation

MSM signals consist of n subcarriers where the i'th subcarrier is modulated by c_i. Therefore, the MSM signal can be written as

$$s(\tau) = \mathrm{Re}\left\{\sum_{i=1}^{n} c_i e^{j\omega_i\tau}\right\} g(\tau) \quad 0 \leq \tau < T, \tag{1}$$

where $g(t)$ is the transmit pulse shape, T is the symbol duration, and ω_i's are carrier frequencies. In this letter as in [2], we consider rectangular pulse shape, i.e. $g(\tau) = u(\tau) - u(\tau - T)$, and dense packing for the subcarriers which implies that $\omega_i = i\frac{2\pi}{T}$ for $i = 1, \ldots, n$. To simplify the mathematical formulation, we normalize the time axis by T to get,

$$s(t) = \mathrm{Re}\left\{\sum_{i=1}^{n} c_i e^{j2\pi i t}\right\} \qquad 0 \leq t \leq 1. \tag{2}$$

To make the transmitted signal positive, we have to add a signal $k(t)$ and guarantee that $k(t)+s(t)$ is non-negative. Generally, for each codeword $(c_1, \ldots, c_n)$, $k(t)$ consists of a dc part k_0 which is equal to $-\min\{s(t) + k_1(t)\}$, and a zero mean part $k_1(t)$. Since we require that the average of $s(t)$ and $k_1(t)$ be zero, the average optical power will be proportional to k_0.

In this letter instead of maximizing the minimum of $s(t) + k_1(t)$ to minimize the average optical power, we consider a stronger condition and we minimize the maximum value of $|\sum_{i=1}^{n} c_i e^{j2\pi t} + b(t)|$ where $k_1(t) = \mathrm{Re}\{b(t)\}$ and $b(t)$ has zero average. This condition limits the variations of the signal both in the negative and positive sides. From a practical point of view, this restriction on the envelope rather than the real part of the signal, will both reduce the average power requirement and eliminate high instantaneous intensity peaks from the transmitted signal.

3 Nehari's Problem and Its Implications on the Reserved Carrier Method

A well-known method in OFDM systems is to use optimized reserved carriers placed at frequencies larger than n to minimize the ratio of the peak to average power of the multicarrier signal [4, 2]. However, here for MSM optical signals, we just need to maximize the minimum of the signal over the amplitude of the reserved carriers.

We can formulate the problem as the following: Let c_i's be given and define $T(z) = \sum_{i=1}^{n} c_i z^{-i}$ and $B(z) = \sum_{i=n+1}^{M} b_i z^{-i}$ where $z = e^{-j2\pi t}$. We would like to find the optimum values of b_i's such that $\|T(z) + B(z)\|_\infty$ is minimized. This also raises the question of how much reduction can we get on $\|T(z) + B(z)\|_\infty$ by using as many reserved carrier as we want, in other words, given the c_i's what is the best γ such that, we have $\|\sum_{i=1}^{n} c_i z^{-i} + \sum_{i=n+1}^{\infty} b_i z^{-i}\|_\infty \leq \gamma$.

A slightly different problem in functional analysis, known as Nehari's problem, states the following result:

Theorem 1. *(Nehari's Theorem [5]) Let $T_1(z) = \sum_{i=1}^{n} c_{n-i+1} z^i$ be an anti-causal function. Then,*

$$\inf_{\text{causal} K_1(z)} \|T_1(z) - K_1(z)\|_\infty = \sigma(\mathcal{H}_T) \tag{3}$$

where $K_1(z) = \sum_{i=0}^{\infty} d_i z^{-i}$ and $\mathcal{H}_T$ is the Hankel operator,

$$\mathcal{H}_T = \begin{bmatrix} c_n & c_{n-1} & \dots & c_1 \\ c_{n-1} & \dots & c_1 & 0 \\ \vdots & & & \\ c_1 & 0 & \dots & \end{bmatrix} \tag{4}$$

and $\sigma(\cdot)$ is the maximum singular value of its argument.

It is worth mentioning that Nehari also showed that the optimum sequence d_i exponentially decreases in i. Thus, in practice, we can truncate the infinite series $K_1(z)$ to its first $M - n$ coefficients. To find the solution to Nehrai's problem, we can use Theorem 12.8.2 of [6] to find $K_1(z)$ numerically. The computation is straightforward and the details can be found in [7].

Using Theorem 1, we can states a lower bound for the maximum average power reduction by using reserved carriers at the end of the signal. Substituting $T_1(z) = z^{n+1} T(z)$ in (3), for all z over the unit circle and any codeword $(c_1, \dots, c_n)$, we get,

$$\sigma(\mathcal{H}_T) = \inf_{\text{causal} K_1(z)} \|T_1(z) - K_1(z)\|_\infty = \inf_{d_0, d_1, \dots} \| \sum_{i=1}^{n} c_i z^{-i} - \sum_{i=n+1}^{\infty} d_{i-n-1} z^{-i} \|_\infty \leq \| \sum_{i=1}^{n} c_i z^{-i} + \sum_{i=n+1}^{M} b_i z^{-i} \|_\infty \tag{5}$$

for any sequence b_i's and any M. Therefore given the c_i's, the maximum average power reduction by placing reserved carriers at the end of the signal is bounded below by $\sigma(\mathcal{H}_T)$ where $\mathcal{H}_T$ is as defined in (4).

For simplicity let's assume c_i's are chosen from a MPSK constellation, i.e. $c_i \in \{e^{j2\pi k/M} : k = 0, \dots, M -$

$1\}$. We can easily bound the maximum singular value of $\mathcal{H}_T$ over all the possible codewords by using the fact that the sum square of the eigenvalues of a matrix is equal to the Frobenius norm of the matrix. This implies that,

$$
\begin{aligned}
\max_{(c_1,\ldots,c_n)} \sigma(\mathcal{H}_T) \leq \max_{(c_1,\ldots,c_n)} \sum_{i=1}^{n} \lambda_i(\mathcal{H}_T^*\mathcal{H}_T) &= \max_{(c_1,\ldots,c_n)} tr\left(\mathcal{H}_T^*\mathcal{H}_T\right) \\
&= \max_{(c_1,\ldots,c_n)} \sum_{i=1}^{n} \sum_{j=1}^{i} |c_i|^2 \\
&= \left\{\sum_{i=1}^{n} i\right\}^{1/2} = \sqrt{\frac{n(n+1)}{2}} \qquad (6)
\end{aligned}
$$

where $\lambda_i(\cdot)$ and $tr(\cdot)$ denote the i'th eigenvalue and trace of its argument, respectively. A lower bound on $\sigma(\mathcal{H}_T)$ can be also obtained by considering the codeword $(1, 1, \ldots, 1)$. The lower and upper bounds are shown in Fig. 1 for n from 10 to 500. Clearly the bounds are very close and therefore the worst case improvement is just from n to $\sqrt{n(n+1)/2}$.

However we can also evaluate the average and the distribution of the maximum singular value of $\mathcal{H}_T$ by generating sufficiently large number of random codewords. Fig. 2 shows the average of $\max \sigma(\mathcal{H}_T)$ for BPSK evaluated by using 10^4 randomly generated codewords. It can be observed that the average behaves very close to $\sqrt{2n \log\log n}$ that is much better than $\sqrt{n \log n}$ which is the average of $\|T(z)\|_\infty$ before using dummy tones asymptotically [8]. Fig. 3 also shows the distribution function of the maximum before using the reserved carriers and the distribution of $\sigma(\mathcal{H}_T)$ which is the best reduction in the maximum that can be achieved by using the Nehari solution $K_1(z)$ as in (5). We also consider the effect of truncating $K_1(z)$ to a finite degree polynomial. Clearly, we have quite a lot of improvement with 20 reserved carriers and further increasing the number of dummy tones does not improve the distribution as much as before.

It is worth noting that the problem of finding the optimum values for a finite number of reserved carriers can be exactly solved by invoking the Bounded Real Lemma and using the LMI (Linear Matrix Inequality) optimization [9, 6](Theorem 12.6.6). However, it can be shown that the complexity of the LMI is $O((n+L)^6)$

where L is the number of reserved tones. On the other hand, all that the computation of $K_1(z)$ requires is a matrix inversion and a maximum singular value computation which has $O(n^3)$ complexity [7]. Therefore truncating the Nehari solution to finite length L seems to be a more practical way of finding good values for reserved carriers.

4 Conclusions

In this letter we consider the achievable average power reduction for multiple subcarrier IM/DD optical signals by using dummy tones with optimum value at the end of the signal. Several problems remain to be answered. We considered the problem of reserved carriers when they are placed at frequencies larger than n. It would be also interesting to investigate the effect of changing their place on the average power reduction. As the numerical results in Fig. 2 suggest, it is intriguing to show that the average of the maximum singular value of random BPSK Hankel matrices is equal to $\sqrt{2n \log \log n}$ with high probability.

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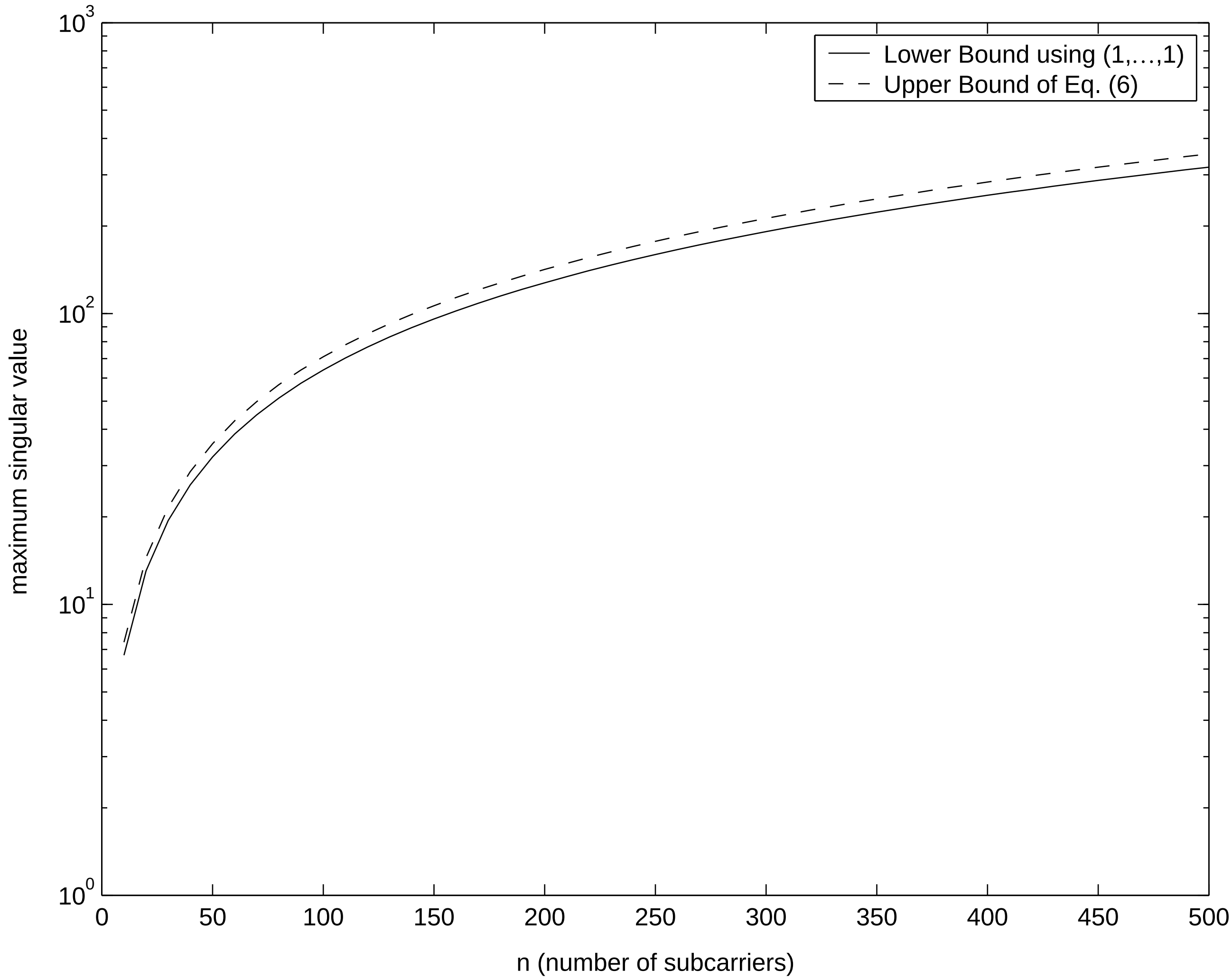

Figure 1: Lower and Upper bounds on the maximum singular value of the random MPSK Hankel matrix.

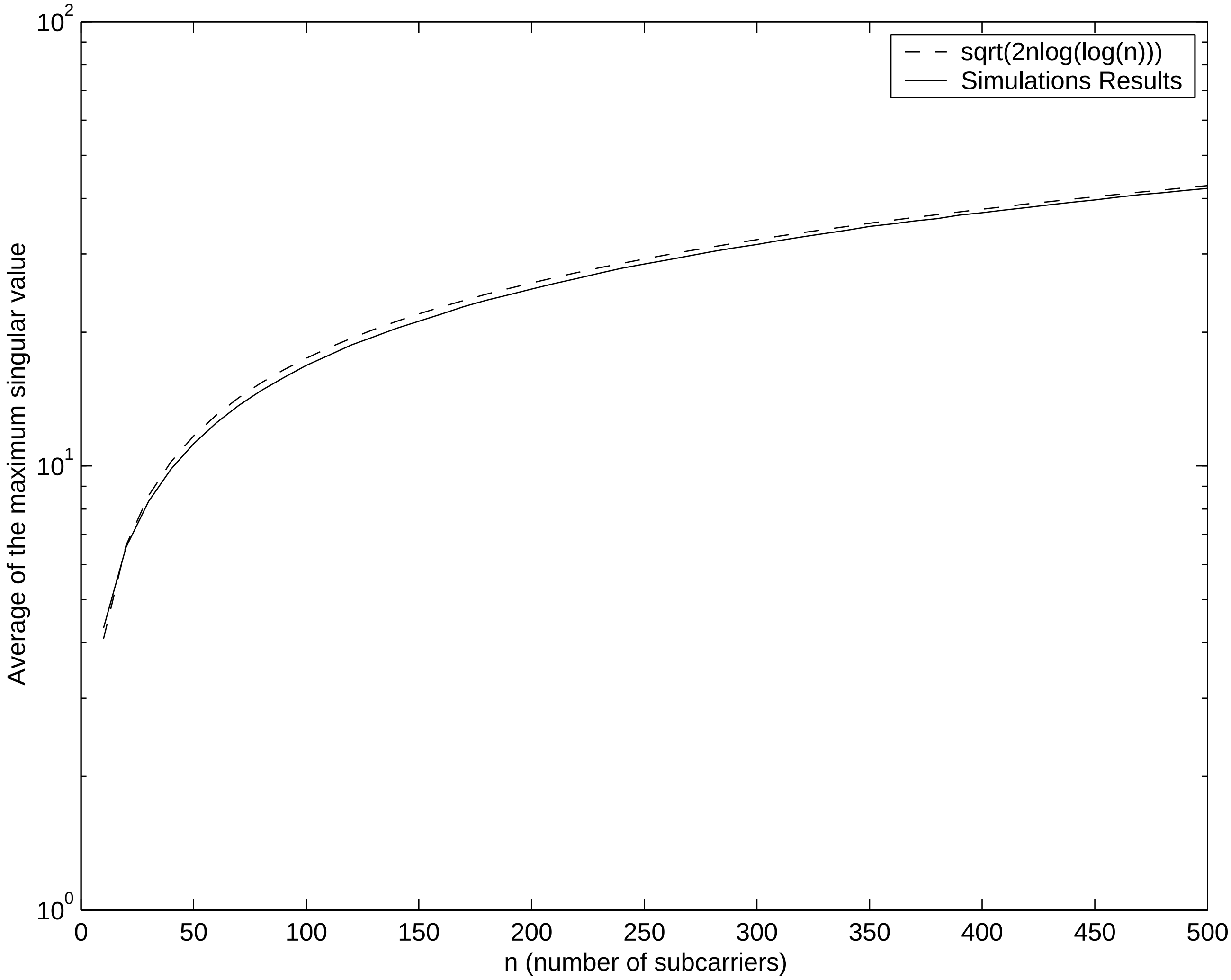

Figure 2: Average of $\sigma(\mathcal{H}_T)$ for BPSK evaluated by using 10^4 randomly generated codewords and $\sqrt{2n \log \log n}$.

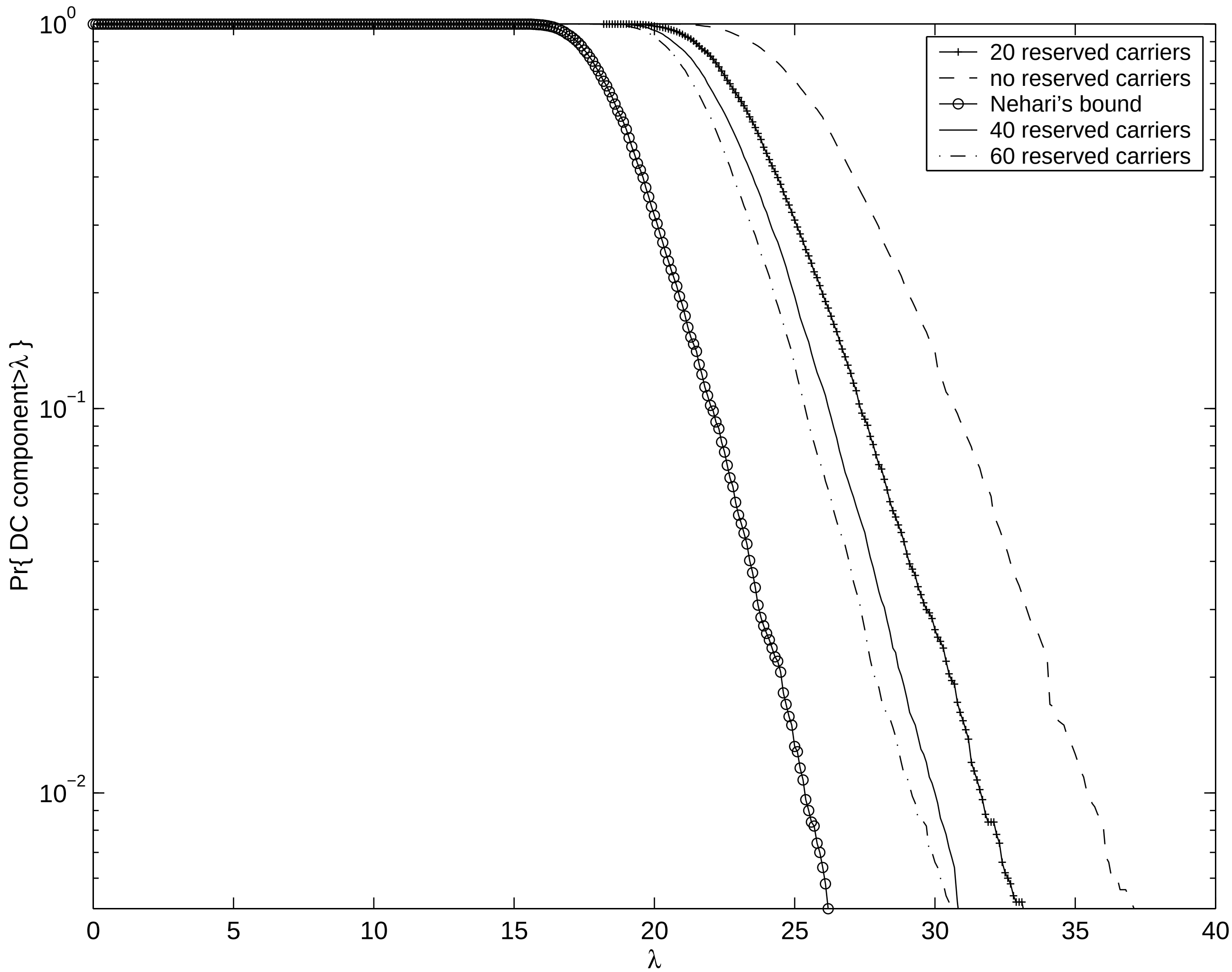

Figure 3: Distribution function of the required dc component with and without using reserved carriers and the effect of truncation of $K_1(z)$ for $n = 128$ using 10^3 randomly generated codewords.