POWER BANDWIDTH TRADE-OFF IN SENSORY AND AD-HOC WIRELESS NETWORKS

AMIR F. DANA   BABAK HASSIBI

California Institute of Technology
Department of Electrical Engineering, MC 136-93
Pasadena, CA 91125, USA
{dana,hassibi}@systems.caltech.edu

ABSTRACT

In this paper we look at the power bandwidth trade-off in sensory and ad-hoc wireless networks under a specific protocol for communication. We consider sensory and ad-hoc wireless networks with n relay nodes and $r \le \sqrt{n}$ source/destination pairs. We assume that the bandwidth resource is unlimited and unconstrained. We show that the minimum power required for maintaining a given scaling law of $R_{sum} = \Theta(f(n))$ for the sum-rate in the network, scales like $\Theta(\frac{f(n)}{\sqrt{n}})$. We also show that the required bandwidth B for achieving this minimum power is finite and is of order $\Theta(f(n))$. It is also proved that the minimum achievable energy per information bit $(\frac{E_b}{N_0})_{min}$, when both the power and the bandwidth resource are unlimited, scales as $\Theta(\frac{1}{\sqrt{n}})$. Furthermore, in this case(unlike the point to point communication systems) the spectral efficiency is non-zero and is of order constant. Finally we show that one does not require infinite bandwidth or infinite power for achieving the scaling law of $(\frac{E_b}{N_0})_{min}$ and as long as the total transmit power *per unit bandwidth* in the network scales like $\frac{1}{\sqrt{n}}$, this scaling law is achievable.

1. INTRODUCTION

In recent years there has been great interest in the capacity of wireless networks. The groundbreaking work of [1] shows that the capacity of a wireless network with n nodes grows at least as $\Theta(\sqrt{n})$. This result is discouraging from the practical point of view because it indicates that the per-user capacity goes to zero like $\frac{1}{\sqrt{n}}$ as the number of nodes in the network increases. Therefore forming large networks will not give us favorable scaling in terms of the achievable rate.

In [2], we have looked at wireless networks from the power consumption point of view. We considered a wireless network with n relay nodes and $r \le \sqrt{n}$ source/destination pairs and a given constant bandwidth for communication. A simple protocol, called "listen and transmit" protocol, is proposed that can achieve a constant per-user rate (as of point to point communication systems) but with only $\frac{1}{\sqrt{n}}$ of the transmit power required in the point to point communication between source/destination pairs for maintaining the same constant rate. Thus forming a wireless network is beneficial in terms of power consumption.

More specifically, consider an AWGN complex channel with transmit power of p, available bandwidth of B and noise spectral level of $\frac{N_0}{2}$. Then we know from [3] that for this channel, the maximum achievable rate is given by

$$R = B\log(1 + \frac{p}{N_0 B})$$

It is also known [3] that the minimum needed transmit energy per information bit, $(\frac{E_b}{N_0})_{min}$, defined as the minimum of the ratio between the transmit power p and the rate R, is $\ln 2$ and is achieved in the low SNR regime. This means that for a fixed bandwidth we can achieve $(\frac{E_n}{N_0})_{min}$ when the transmit power goes to zero, i.e., $p \to 0$. On the other hand, for a fixed and given power, we achieve the minimum energy per bit, $(\frac{E_b}{N_0})_{min}$, when we are in the wideband regime ($B \to \infty$). From another point of view, in AWGN channels, the minimum needed energy per information bit $(\frac{E_b}{N_0})_{min}$ is achieved with zero spectral efficiency, C. Therefore there is some trade-off between power and bandwidth (or energy per bit and spectral efficiency equivalently) in AWGN point to point channel. Verdú has looked at this trade-off for a number of communication systems in [5] and has derived some conditions for optimality of the signaling in the wideband regime.

Motivated by these ideas in the point to point communication systems, one important and interesting problem in the network setup is to determine the fundamental limits of $\frac{E_b}{N_0}$ and also the trade-off between $\frac{E_b}{N_0}$ and C (or power and bandwidth equivalently) in wireless networks. This problem is still an unsolved problem.

The result of [2] shows that for wireless sensory and ad-hoc networks and for a fixed bandwidth available, by using the "listen and transmit" protocol and by allocating the transmit power at the source nodes and the relay nodes appropriately, we can achieve a energy per information bit of

This work was supported in part by the National Science Foundation under grant no. CCR-0133818, by the office of Naval Research under grant no. N00014-02-1-0578, and by Caltech's Lee Center for Advanced Networking.

order $\frac{1}{\sqrt{n}}$.

In this paper we have a different and more general approach. We assume that the bandwidth resource is unlimited and we allow for arbitrary bandwidth allocation. We consider the same "listen and transmit" protocol of [2]. The goal is to analyze the trade-off between power and bandwidth in this protocol. Namely, we are interested in following questions:

- What is the minimum power required for maintaining a given rate (or a scaling law for rate)?
- What is the minimum achievable energy per bit $(\frac{E_b}{N_0})_{min}$ and the corresponding spectral efficiency C, when we are allowed to do bandwidth and power allocation?
- For what range of power P_t, and bandwidth B, the optimal scaling law of the minimum energy per bit is achievable?

In this paper we provide answer to the above questions.

For a given desired rate, it is shown that the minimum needed transmit power is of order $\Theta(\frac{1}{\sqrt{n}})$ and the required bandwidth is of order $\Theta(1)$. We also analyze the minimum required power when we are interested , not in a constant rate, but in a rate that scales with the size of the network.

We also show that the minimum of energy per bit $(\frac{E_b}{N_0})_{min}$, using "listen and transmit" protocol, over all the power and bandwidth allocations, scales as $\Theta(\frac{1}{\sqrt{n}})$. The corresponding per-user spectral efficiency C is also of order $\Theta(1)$. This result is interesting in the sense that using this protocol we can obtain a small energy per bit for large networks while keeping C constant.

It is also proved that as long as the total transmit power per unit bandwidth, i.e., $\frac{P_t}{B}$, scales like $\Theta(\frac{1}{\sqrt{n}})$ ($\Theta(\frac{r}{\sqrt{n}})$) for sensory(ad-hoc) networks, one can achieve the scaling law of minimum energy per bit $(\frac{E_b}{N_0})_{min}$ and the constant per-user spectral efficiency C. Therefore, we can achieve with $(\frac{E_b}{N_0})_{min}$ with *finite* bandwidth and transmit power.

This paper is organized as follows. In the next section, we introduce the network model for sensory and ad-hoc wireless networks. In Section 3, the trade-off between the power and bandwidth for sensory networks is analyzed. In Section 4, we consider the bandwidth power trade-off for ad-hoc wireless networks. Section 5 is the conclusion.

2. NETWORK MODEL

In this paper we will consider *Ad-hoc* wireless networks with r simultaneous source/destination pairs and n relay nodes located in a domain of fixed area A. (See Fig. 1.) The relay nodes are for aid of communication between sources and destinations. We assume that $r \leq \sqrt{n}$. The special case when there is only one source/destination pair, i.e. $r = 1$, corresponds to the *Sensory wireless networks.* (See Fig. 2.)

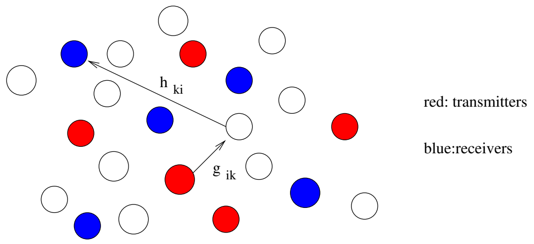

Fig. 1. An ad-hoc network.

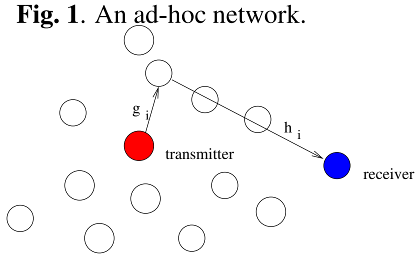

Fig. 2. A sensory network.

Transmit powers at every source node is assumed to be equal to p. The relay power, for all the relay nodes is assumed to be the same and is denoted by σ_r^2. The bandwidth is denoted by B. The noise introduced in every reception is denoted by $\mathcal{CN}(0, \sigma_n^2)$ and is assumed to be an additive white circularly-symmetric Gaussian random variable with zero mean and spectral level of $\frac{N_0}{2}$ and therefore a variance of $\sigma_n^2 = N_0 B$. The channel coefficient from source i ($i = 1, \ldots, r$) to relay node j ($j = 1, \ldots, n$) is denoted by g_{ij} and from relay node j to destination node k ($k = 1, \ldots, r$) is denoted by h_{jk}, where the $\{g_{ij}, h_{jk}\}$ for all i, j, k are i.i.d [1] complex-valued random variables with zero mean, unit variance and fourth order moment κ. We assume that all the nodes know their local connections, but not the remaining connections in the network. In other words, node j knows all the connections $\{g_{ij}, i = 1, \ldots, r\}$ and $\{h_{jk}, k = 1, \ldots, r\}$.

We shall define two quantities which we will use later on in this paper. The transmitted energy per information bit for a network (normalized to the background noise spectral level) is denoted by $\frac{E_b}{N_0}$ and is defined as

$$\frac{E_b}{N_0} = \frac{P_t}{N_0 R_{sum}} \tag{1}$$

where P_t is the total power consumed for transmission. R_{sum} is the achievable sum-rate: $R_{sum} = \sum_{i=1}^{r} R_i$, where R_i is the rate of information transmission for source/destination pair i. This quantity shows the efficiency of the communication in terms of power consumption.

The minimum energy per information bit is denoted by $(\frac{E_b}{N_0})_{\min}$ and is equal to the minimum value of the ratio in

[1] Although we assume the channels to be identically distributed and hence ignore the geometry of the network, the ideas in this paper carry over to the case when the statistics of the channel coefficients depend on the geometry of the network. For more information see [2]

the right hand side of equation (1), when we have freedom in choosing our resources i.e. transmit and relay powers ps and bandwidth B.

The spectral efficiency, C, is defined as the ratio between the data rate R_{sum} and the bandwidth B,i.e.

$$\mathsf{C} = \frac{R_{sum}}{B} \quad (2)$$

This quantity, as its name suggests, shows how efficient the spectrum is used in communication.

3. SENSORY NETWORKS

In this section we analyze the trade-off between power and bandwidth for sensory wireless networks under the simple, so called "listen and transmit" protocol.

3.1. An achievable rate

In this, so-called "listen and transmit" protocol, communication is done in two intervals:

1. **Listen interval:** In this interval the transmitter sends the data and the relay nodes only listen. Relay node i receives $r_i = g_i s + v_i$, where v_i is $\mathcal{CN}(0, \sigma_n^2)$.

2. **Transmit interval:** Each node, using its knowledge of the local connections, transmits a scaled version of the signal it has received in the first interval:

$$t_i = \frac{\sigma_r}{\sqrt{\kappa p + \sigma_n^2}} g_i^* h_i^* r_i \qquad i = 1, 2, \ldots, n. \quad (3)$$

The scalar is chosen so that the signal component coherently adds in the receiver and so that on the average, relay nodes transmit with power σ_r^2.

The received signal at the receiver is:

$$\begin{aligned} y &= \sum_{i=1}^{n} h_i t_i + w \quad (4) \\ &= \underbrace{\sum_{i=1}^{n} \frac{\sigma_r}{\sqrt{\kappa p + \sigma_n^2}} |g_i|^2 |h_i|^2}_{\alpha} s + \underbrace{\sum_{i=1}^{n} \frac{\sigma_r}{\sqrt{\kappa p + \sigma_n^2}} |h_i|^2 g_i^* v_i + w}_{w'} \end{aligned}$$

where w is $\mathcal{CN}(0, \sigma_n^2)$. Now using the same argument as in [2], we can derive lower and upper bounds for the achievable rate using "listen and transmit" protocol. The next lemma, state these bound. The proof is skipped for brevity.

Lemma 1 *The achievable rate R_{sum} for the sensory wireless network defined in Section 2 is bounded as*

$$R_{sum} \geq \frac{B}{2} \log\left(1 + \frac{n^2 \sigma_r^2 p}{N_0^2 B^2 + (N_0 B \kappa + \kappa^2 - 1)(p + n\sigma_r^2)}\right)$$
$$R_{sum} \leq \frac{B}{2} \log\left(1 + \frac{n^2 \sigma_r^2 p + n\sigma_r^2(\kappa^2 - 1)p}{N_0^2 B^2}\right)$$

Now note that for a given total consumed power for transmission $P_t = \frac{1}{2}(n\sigma_r^2 + p)$ ($\frac{1}{2}$ comes from the fact that each node transmits in half of the intervals), the lower bound and the upper bound for R_{sum} in the above expression are maximized when $n\sigma_r^2 = p = P_t$. So we take this point as our operating point for source node and relay node powers and rewrite the above expression in terms of P_t as

$$R_{sum} \geq R_{lower} = \frac{B}{2} \log\left(1 + \frac{nP_t^2}{N_0 B(\kappa P_t + N_0 B) + P_t(\kappa^2 - 1)}\right)$$
$$R_{sum} \leq R_{upper} = \frac{B}{2} \log\left(1 + \frac{(n + \kappa^2 - 1)P_t^2}{N_0 B(N_0 B + \kappa P_t)}\right) \quad (5)$$

3.2. Power Bandwidth Trade-off: Sensory Network

In this section we look at the minimum power needed for achieving a given rate in the network, when the bandwidth resource is unconstrained.

We start by finding the scaling behavior of the maximum achievable rate of sensory networks using "listen and transmit" protocol, for different values of $P_t(n)$ (i.e., in the way P_t scales with n) when the bandwidth resource is unlimited. Denote this rate by $R_{sum}^{\star}$. First we start by maximizing R_{lower}. Using (5), by defining $x = \frac{\sqrt{n}P_t}{N_0 B}$, we have

$$R_{lower} = \frac{\sqrt{n}P_t}{2N_0} \max_x \frac{1}{x} \log(1 + \frac{nP_t}{\frac{\sqrt{n}P_t}{x^2}(\sqrt{n} + \kappa x) + \kappa^2 - 1}) \quad (6)$$

Looking at R_{lower}, by keeping only the first term in the denominator we have

$$R_{lower} \leq \frac{\sqrt{n}P_t}{2N_0} \underbrace{\frac{1}{x} \log(1 + x^2)}_{\leq c_1} \leq \frac{c_1 \sqrt{n} P_t}{2N_0} \quad (7)$$

We can derive another bound for R_{lower}. For this define $z = \frac{\sqrt{P_t}}{N_0 B}$. Then by writing equation (5) in terms of z we have

$$R_{lower} \leq \frac{\sqrt{P_t}}{2N_0} \frac{1}{z} \log(1 + \frac{nP_t}{\kappa^2 - 1 + \frac{1}{z}(\kappa\sqrt{P_t} + \frac{1}{z})}) \quad (8)$$

Now by ignoring the middle term in the denominator and using the fact that $\ln(1 + x) \leq x$ for every positive x, we get

$$R_{lower} \leq \frac{nP_t\sqrt{P_t}}{2N_0 \ln 2} \frac{z}{(\kappa^2 - 1)z^2 + 1} \leq \frac{nP_t\sqrt{P_t}}{4N_0 \ln 2\sqrt{\kappa^2 - 1}} \quad (9)$$

where we have used the fact that the maximum value of $\frac{z}{(\kappa^2-1)z^2+1}$ is when $z = \frac{1}{\sqrt{\kappa^2-1}}$. Putting (7) and (9) together yields

$$R_{lower} \leq \min\left\{\frac{nP_t\sqrt{P_t}}{4N_0 \ln 2\sqrt{\kappa^2 - 1}}, \frac{c_1\sqrt{n}P_t}{2N_0}\right\} \quad (10)$$

Also by considering the two cases when $\lim_{n\to\infty} nP_t = 0$ and $\lim_{n\to\infty} nP_t > 0$ separately, and setting $x = 1$ and $z = 1$ in equations (6) and (8), it can be shown that R_{lower} actually scales as the right hand side of (10). After some manipulation we have

$$R_{lower} = \begin{cases} \Theta(n\sqrt{P_t}P_t) & \text{if } \lim_{n\to\infty} nP_t = 0, \\ \Theta(\sqrt{n}P_t) & \text{if } \lim_{n\to\infty} nP_t > 0. \end{cases} \tag{11}$$

Note that the required bandwidth in these two cases is $B = \Theta(\sqrt{P_t})$ and $B = \Theta(\sqrt{n}P_t)$ respectively. We can do the same analysis for the upper bound. In this case, it can be verified that

$$R_{upper} = \Theta(\sqrt{n}P_t) \tag{12}$$

Now putting (11) and (12) together, we have

$$\begin{aligned} R^{\star}_{sum} = \Theta(\sqrt{n}P_t) \qquad & \text{if } \lim_{n\to\infty} nP_t > 0 \\ \Theta(n\sqrt{P_t}P_t) \le R^{\star}_{sum} \le \Theta(\sqrt{n}P_t) \quad & \text{otherwise} \end{aligned} \tag{13}$$

We can interpret the above discussion in terms of minimum power required for maintaining a desired rate. We have stated this in the theorem below.

Theorem 1 *Consider a sensory wireless network with n relay nodes and one source/destination pair, and the reception noise and channel model as described in Section 2. Assume that we are using the "listen and transmit" protocol. Also assume that the bandwidth resource is unlimited and unconstrained. Then the minimum power required for attaining a given scaling law for the rate $R_{sum} = \Theta(f(n))$, where $\lim_{n\to\infty} \sqrt{n}f(n) > 0$, is given by*

$$P_t = \Theta(\frac{f(n)}{\sqrt{n}})$$

and the required bandwidth B for achieving this minimum power is some finite value of order $\Theta(f(n))$.

Remark 1: If $\sqrt{n}R_{sum}$ grows slower than constant in terms on n, we can still give a bound on the minimum required power for attaining R_{sum}. In this case we will have

$$\Theta(\frac{R_{sum}}{\sqrt{n}}) \le P_t \le \Theta((\frac{R_{sum}}{n})^{\frac{2}{3}})$$

Remark 2: It can be seen from (5) that for $B \to \infty$ the achievable rate goes to zero.

3.3. $\frac{E_b}{N_0}$ versus C

In this section, we will look at the trade-off between the energy per bit of information $\frac{E_b}{N_0}$ and the spectral efficiency C under the "listen and transmit" protocol.

We aim to find the minimum energy per bit of information $(\frac{E_b}{N_0})_{min}$. For this, we maximize the energy per information bit when there is no constraint on power and bandwidth. By defining $x = \frac{\sqrt{n}P_t}{BN_0}$ and looking at the lower bound in (5), we can write

$$(\frac{E_b}{N_0})_{min} \le \frac{1}{\sqrt{n}} \min_{x,B} \frac{2x}{\log\left(1 + \frac{x^2 BN_0}{N_0 B(x\frac{\kappa}{\sqrt{n}}+1) + \frac{(\kappa^2-1)}{\sqrt{n}}x}\right)} \tag{14}$$

It is clear from the above equation that the minimum of the right-hand side occurs for $B \to \infty$, i.e. when we are in wideband regime. So we can simplify the above equation further to

$$(\frac{E_b}{N_0})_{min} \le \frac{1}{\sqrt{n}} \min_{x} \frac{2x}{\log\left(1 + \frac{x^2}{x\frac{\kappa}{\sqrt{n}}+1}\right)} \tag{15}$$

It can be verified that the minimum value and the corresponding value of x in the above minimization problem are bounded by constant values for all n. Therefore, by plugging $x = 1$ in the above equation, we will get the following bound for $(\frac{E_b}{N_0})_{min}$ which is tight for large ns

$$(\frac{E_b}{N_0})_{min} \le \frac{1}{\sqrt{n}} \frac{2}{\log\left(1 + \frac{1}{\frac{\kappa}{\sqrt{n}}+1}\right)} \tag{16}$$

using the same argument as above for the upper bound in (5), we can show that for large n, $(\frac{E_b}{N_0})_{min} \ge \alpha\frac{1}{\sqrt{n}}$ for some positive constant α. Therefore, using (16), it becomes clear that for large n,

$$(\frac{E_b}{N_0})_{min} = \Theta(\frac{1}{\sqrt{n}})$$

and this is achieved for $B \to \infty$ (i.e., the wideband regime) and $\frac{p}{N_0 B} = \frac{n\sigma_r^2}{N_0 B} = \frac{1}{\sqrt{n}}$. In this case, the spectral efficiency is equal to

$$\mathsf{C} = \frac{1}{2}\log(1 + \frac{1}{\frac{\kappa}{n}+1}) = \Theta(1)$$

Thus for large networks, the spectral efficiency corresponding the minimum energy per information bit is some positive constant. Note that in the point to point communication, the minimum energy per information bit is achieved when the spectral efficiency is zero [5], which is not the case in here.

In the above analysis, $(\frac{E_b}{N_0})_{min}$ (for any n) was achieved for the case when $B \to \infty$ and $\frac{P_t}{N_0 B} = \frac{1}{\sqrt{n}}$. However, for large networks (n large enough) we can get the same asymptotic of $\Theta(\frac{1}{\sqrt{n}})$ for $\frac{E_b}{N_0}$ by using finite bandwidth, B, and finite power, P_t. We have stated this in Theorem 2. We skip the proof for brevity.

Theorem 2 *Consider a sensory wireless network with n relay nodes and one source/destination pair, and the reception* *noise and channel model as described in Section 2. Assume that we are using the "listen and transmit" protocol. Also assume that the bandwidth and transmit power resources are unlimited and unconstrained. Then the minimum energy per information bit achievable with this protocol and the corresponding spectral efficiency scale as* $\Theta(\frac{1}{\sqrt{n}})$ *and* $\Theta(1)$ *respectively, i.e.,*

$$(\frac{E_b}{N_0})_{min} = \Theta(\frac{1}{\sqrt{n}}) \quad \mathsf{C} = \Theta(1) \tag{17}$$

furthermore any power and bandwidth allocation that satisfies

$$B \geq \frac{K}{\sqrt{n}} \quad P_t = \frac{\alpha B N_0}{\sqrt{n}}$$

for some positive constant K *and* α*, achieves the scalings of (17).*

4. AD-HOC NETWORKS

We now analyze power bandwidth trade-off under an extension of the "listen and transmit protocol" for ad-hoc networks. As mentioned earlier we assume that the number of transmit/receive pairs r satisfies $r \leq \sqrt{n}$.

4.1. An achievable rate

In this section we find an achievable rate for ad-hoc networks using the "listen and transmit" protocol. This protocol is performed as follows:

1. **Listen interval:** Each of the r transmit users transmit the signal s_i. All other nodes listen. Relay node j, receives $r_j = \sum_{i=1}^{r} g_{ij} s_i + v_j$.

2. **Transmit interval:** Each relay node j, transmits, t_j, a scaled version of what it has previously received $t_j = d_j r_j$, where the scalar, d_j, depends only on local knowledge of the channel coefficients:

$$d_j = \frac{\sigma_r \left(\sum_{i=1}^{r} g_{ij}^* h_{ji}^* \right)}{\sqrt{r\sigma_n^2 + r(r+\kappa-1)p}}$$

   It can be shown that with this choice of d_j, the average transmit power for relay nodes is σ_r^2.

The received signal at receiver k is

$$\begin{aligned} y_k &= \frac{\sigma_r}{\sqrt{r\sigma_n^2 + r(r+\kappa-1)p}} \sum_{j=1}^{n} h_{jk} t_j + w_k \\ &= \sum_{i=1}^{r} \Bigg(\underbrace{\frac{\sigma_r \sum_{j=1}^{n} \sum_{l=1}^{r} (g_{ij} g_{lj}^* h_{jl}^* h_{jk})}{\sqrt{r\sigma_n^2 + r(r+\kappa-1)p]}}}_{\alpha_i^k} \Bigg) s_i + \\ & \sum_{j=1}^{n} \Bigg(\underbrace{\frac{\sigma_r \sum_{l=1}^{r} g_{lj}^* h_{jl}^* h_{jk}}{\sqrt{r\sigma_n^2 + r(r+\kappa-1)p}}}_{\beta_j^k} \Bigg) v_j + w_k \end{aligned} \tag{18}$$

Now using the same techniques as in [2] we can find upper and lower bounds for the mutual information between y_k and s_k, denoted by R_k. Therefore we can find a bound on the achievable sum-rate, $R_{sum} = \sum_{k=1}^{r} R_k$, in the network. We have stated these bounds in the lemma below.

Lemma 2 *Consider a wireless ad-hoc network with* n *relay nodes and* $r \leq \sqrt{n}$ *source/destination pair and the same model as described in Section 2. Furthermore assume that* $n >> r >> 1$*. Then the achievable sum-rate* R_{sum} *using the "listen and transmit" protocol satisfies*

$$\begin{aligned} R_{sum} \geq & \frac{rB}{2} \log\big(1 + \frac{rpn^2\sigma_r^2}{r^2(N_0 B + rp)(N_0 B + n\sigma_r^2)}\big) \\ R_{sum} \leq & \frac{rB}{2} \log\big(1 + \frac{rpn^2\sigma_r^2}{r^2 N_0^2 B^2}\big) \end{aligned} \tag{19}$$

Similar to the sensory case, for a total transmitted power of $P_t = \frac{1}{2}(rp + n\sigma_r^2)$ ($\frac{1}{2}$ comes from the fact that each node transmits only half of the times), the upper and lower bounds are maximized for $rp = n\sigma_r^2 = P_t$. Therefore, we choose these powers and rewrite (19) as

$$\begin{aligned} R_{sum} \geq & \frac{rB}{2} \log\big(1 + \frac{nP_t^2}{r^2(N_0 B + P_t)^2}\big) \\ R_{sum} \leq & \frac{rB}{2} \log\big(1 + \frac{nP_t^2}{r^2 N_0^2 B^2}\big) \end{aligned} \tag{20}$$

4.2. Power Bandwidth Trade-off : Ad-hoc networks

In this section we consider the trade-off between the scaling behavior of R_{sum}, P_t and B for large ad-hoc networks. Similar to the sensory case, we start by finding the maximum achievable sum-rate using the "listen and transmit" protocol, for different values of $P_t(n)$ (i.e., in the way it scales with n), when the bandwidth resource is unlimited. We denote this maximum achievable rate by $R_{sum}^{\star}(P_t)$.

By defining $x = \frac{\sqrt{n} P_t}{r N_0 B}$, we can write the maximum of the lower bound in (20), in terms of x, as below

$$R_{sum}^{\star}(P_t) \geq \frac{\sqrt{n} P_t}{2N_0} \max_x \frac{1}{x} \log(1 + \frac{x^2}{(1 + \frac{r}{\sqrt{n}} x)^2}) \tag{21}$$

Note that the maximization problem only depends of n and r. Let $f(n,r)$ be this maximum value and $g(n,r)$ be the corresponding maximizing x. Then we have

$$\begin{aligned} R_{sum}(P_t)^{\star} &\geq \sqrt{n} f(n,r) \frac{P_t}{2N_0} \\ B &= \frac{\sqrt{n} P_t}{N_0 r g(n,r)} \end{aligned} \tag{22}$$

Now since $r \leq \sqrt{n}$, it can be easily verified that $f(n,r)$ and $g(n,r)$ are always bounded by two constant real numbers, i.e.

$$k_1 \leq f(n,r) \leq k_2 \quad c_1 \leq g(n,r) \leq c_2 \tag{23}$$

for positive constant k_i, $c_i \quad i = 1, 2$. Also using the upper bound in (20), we can show that for large networks (i.e., large n and r), we can not get a better scaling for the rate than the one achieved in (22). Therefore for large ad-hoc networks, for any $P_t(n)$

$$R^{\star}_{sum}(P_t) = \Theta(P_t(n)\sqrt{n})$$

and the required bandwidth in this case is $B = \Theta(\frac{\sqrt{n}P_t(n)}{r})$.

We can interpret the above result in the following way as well. For a given scaling behavior of R_{sum}, the minimum transmit power that is required to maintain that rate behave like $P_t = \Theta(\frac{R_{sum}}{\sqrt{n}})$. The required bandwidth is also $B = \Theta(\frac{R_{sum}}{r})$. We have summarized the above discussion in the following theorem. We skip the proof for brevity.

Theorem 3 *Consider a wireless ad-hoc network with n relay nodes and $r \leq n$ source/destination pairs. Assume the same noise and channel model described in Section 2. Furthermore assume that the bandwidth resource is unlimited and unconstrained. Then for a given desired scaling law for the sum-rate $R_{sum} = \Theta(f(n))$, the minimum required power for achieving that rate using "listen and transmit" protocol scales as*

$$P_t = \Theta\left(\frac{f(n)}{\sqrt{n}}\right)$$

Also the required bandwidth for attaining that minimum power scaling law scales like $B = \Theta(\frac{f(n)}{r})$.

Remark 1: From (22) we can see that unlike point to point communication systems, in the "listen and transmit" protocol the required bandwidth for maximizing the rate for a given power is a finite quantity given in (22). Furthermore for $B \to \infty$ the rate goes to zero.

From the discussion above and by using (1) and (2) we can state the following theorem about $(\frac{E_b}{N_0})_{min}$ and its corresponding spectral efficiency. We skip the proof for brevity.

Theorem 4 *Consider a wireless ad-hoc network with n relay nodes and $r \leq \sqrt{n}$ source/destination pairs. Assume the same noise and channel model described in Section 2. Then the minimum of transmitted energy per information bit $(\frac{E_b}{N_0})_{min}$ over all the possible power and bandwidth allocations scales like*

$$(\frac{E_b}{N_0})_{min} = \Theta(\frac{1}{\sqrt{n}}).$$

The corresponding spectral efficiency for this case is

$$\mathsf{C} = \Theta(r).$$

Furthermore, these value are achieved for $\frac{rp}{N_0 B} = \frac{n\sigma_r^2}{N_0 B} = \Theta(\frac{r}{\sqrt{n}})$. Thus, as long as the source and relay node power per bandwidth is of this order, we can achieve $(\frac{E_b}{N_0})_{min}$ and spectral efficiency of order $\Theta(r)$.

This result is interesting because it says that in the "listen and transmit" protocol the minimum energy per bit is achieved with per-user spectral efficiency of order constant.

5. CONCLUSION

In this paper, we have addressed the trade-off between power and bandwidth in wireless sensory and ad-hoc wireless networks. We showed that asymptotically, as the number of nodes in the network, n, grows larger, we can achieve a $\frac{E_b}{N_0}$ (i.e. transmit energy per bit of information) of $\Theta(\frac{1}{\sqrt{n}})$ for sensory networks. In addition, it is shown that the same result holds for ad-hoc networks provided that the the number of source/destination pairs is less than $\sqrt{n}$. We further showed that the achievable spectral efficiency for these networks at that $\frac{E_b}{N_0}$ is of order constant per source/destination pair. Finally we saw that the important quantity in achieving this energy per bit is the power per unit bandwidth. As long as the total power per bandwidth is of order $\frac{1}{\sqrt{n}}$ for sensory networks and of order $\frac{r}{\sqrt{n}}$ for ad-hoc networks, we can achieve the optimal scaling for energy per bit.

Currently we are working on the performance of other protocols such as the multi-hop protocol of [6] in terms of spectral efficiency and energy per information bit for wireless networks. Determining the fundamental limits of minimum needed transmit energy per information bit and its trade-off with spectral efficiency for wireless networks is also an important problem which is still very much open.

6. REFERENCES

[1] P. Gupta and P.R. Kumar, "The capacity of wireless networks," *IEEE Trans. Info. Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[2] A. F. Dana and B. Hassibi, "On the power efficiency of sensory and ad-hoc wireless networks," *Submitted to IEEE Trans. Info. Theory*, 2003.

[3] C. E. Shannon, "A mathematical theory of communication," *Bell Sys. Tech. Journal*, pp. 379–423, 623–656, 1948.

[4] Muriel Médard, "The effect upon channel capacity in wireless communications of perfect and imperfect knowledge of the channel," *IEEE Trans. Info. Theory*, vol. 46, no. 3, pp. 933–946, 2000.

[5] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1319–1343, 2002.

[6] G. Kramer, M. Gastpar, and P. Gupta, "Capacity theorems for wireless relay channels," *41st Annual Allerton Conf. on Commun., Control and Comp.*, 2003.