DISTRIBUTED SPACE-TIME CODES IN WIRELESS RELAY NETWORKS

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ABSTRACT

We apply the idea of space-time coding devised for multiple antenna systems to the communication over a wireless relay network. We use a two stage protocol, where in one stage the transmitter sends information and in the other, the relay nodes encode their received signals into a "distributed" linear dispersion code, and then transmit the coded signals to the receiver. We show that for high SNR the *pairwise error probability (PEP)* behaves as $(\log P/P)^{\min\{T,R\}}$, with T the coherence interval, R the number of relay nodes and P the total transmit power. Thus, apart from the $\log P$ factor and assuming $T \geq R$, the system has the same diversity as a multi-antenna system with R transmit antennas, which is the same as assuming that the R relay nodes can fully cooperate and have full knowledge of the transmitted signal. We further show that for a fixed total transmit power across the entire network, the optimal power allocation is for the transmitter to expend half the power and for the relays to collectively expend the other half. We also show that at low and high SNR, the coding gain is the same as that of a multi-antenna system with R antennas. At intermediate SNR, it can be quite different, which has implications for the design of distributed space-time codes.

1. INTRODUCTION

It is known that multiple antennas can greatly increase the capacity and reliability of a wireless communication link in a fading environment using space-time codes [1, 2, 3, 4]. Recently, with the increasing interests in ad hoc networks, researchers have been looking for methods to exploit spatial diversity using the antennas of different users in the network [5, 6, 7, 8, 9]. In [8], the authors exploit spatial diversity using the repetition and space-time algorithms. The mutual information and outage probability of the network are analyzed. However, in their model, the relay nodes need to decode their received signals. In [9], a network with a single relay under different protocols is analyzed and second order spatial diversity is achieved. In [10], the authors use space-time codes based on the Hurwitz-Radon matrices and conjecture a diversity factor around $R/2$ from their simulations. Also, the simulations in [11] show that the use of Khatri-Rao codes lowers the average bit error rate. In this paper, we consider a relay network with fading and apply a linear dispersion space-time code [12] among the relays. The problem we are interested in is: "Can we increase the reliability of a wireless network by using space-time codes among the relay nodes?"

The wireless relay network model we use is similar to those in [13, 14]. In [13], the authors show that the capacity of a network with n nodes behaves like $\log n$. In [14], a power efficiency that behaves like $\sqrt{n}$ is obtained. Both results are based on the assumption that every relay knows its local channels. Therefore, the system should be synchronized at the carrier level. In this paper, we assume that the relays do not know the channel information. All we need is the much more reasonable assumption that the systems is synchronized at the symbol level.

Our work shows that using linear dispersion space-time codes among the relay nodes can achieve a diversity of $\min\{T,R\}\left(1-\frac{\log\log P}{\log P}\right)$. When $T \geq R$, the transmit diversity is linear in the number of relays (size of the network) and is a function of the total transmit power. For very large P ($P \gg \log P$), the diversity is approximately R and the coding gain is $\det(S_i - S_j)^*(S_i - S_j)$ where S_i and S_j are codewords in the distributed space-time code. Therefore, at very high SNR, the same transmit diversity and coding gain are obtained as in the multiple antenna case, which means that the system works as if the relays can fully cooperate and have full knowledge of the transmitted signal.

2. SYSTEM MODEL

We first introduce some notations used in the paper. For a complex matrix A, $\bar{A}$, A^t, and A^* denote the conjugate, the transpose, and the conjugate transpose of A, respectively. $\det A$, $\text{rank}\, A$, and $\text{tr}\, A$ indicate the determinant, rank, and trace of A, respectively. I_n denotes the $n \times n$ identity matrix and 0_{mn} is the $m \times n$ matrix with all zero entries. We often omit the subscripts when there is no confusion. log indicates the natural logarithm. $\|\cdot\|$ indicates the Frobinus norm.

Consider a wireless network with $R + 2$ nodes which are placed randomly and independently according to some distribution. There are 1 transmitter and 1 receiver. All the

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other R nodes work as relays. The transmitter has 1 transmit antenna, the receiver has 1 receive antenna, and every relay node has 1 transmit and 1 receive antenna. Denote the channel from the transmitter to the ith relay as f_i, and the channel from the ith relay to the receiver as g_i. Assume that f_i and g_i are independent complex Gaussian with zero-mean and unit-variance. If relay i knows f_i and g_i, it is proved in [13] and [14] that the capacity behaves like $\log R$ and a power efficiency that behaves like $\sqrt{R}$ can be obtained. In our relay network, we assume that the relay nodes know only the statistical distribution of the channels. However, we assume that the receiver knows all f_i and g_i. Its knowledge of the channels can be obtained by sending training signals from the relays and the transmitter. Our main question is how much PEP improvement can be obtained?

Assume that the transmitter wants to send the signal $\mathbf{s} = [s_1, \cdots, s_T]^t$ in the codebook $\{\mathbf{s}_1, \cdots, \mathbf{s}_L\}$ to the receiver, where L is the cardinality of the codebook. $\mathbf{s}$ is normalized as $\mathrm{E}\,\mathbf{s}^*\mathbf{s} = 1$. The transmission is accomplished by the following two-step strategy. From time 1 to T, the transmitter sends signals $\sqrt{P_1 T}s_1, \cdots, \sqrt{P_1 T}s_T$ to every relay. Based on the normalization of $\mathbf{s}$, The average total transmit power of the T transmissions is $P_1 T$. The received signal at the ith relay at time τ is denoted as $r_{i,\tau}$, which is corrupted by the noise $v_{i,\tau}$. From time $T+1$ to $2T$, the ith relay node transmits $t_{i,1}, \cdots, t_{i,T}$ to the receiver based on its received signals. We denote the received signal at the receiver at time $\tau + T$ by x_τ, and the noise at the receiver at time $\tau + T$ by w_τ. Assume that the noises are complex Gaussian with zero-mean and unit-variance. We use the following notations:

$$\mathbf{v}_i = \begin{bmatrix} v_{i,1} \\ \vdots \\ v_{i,T} \end{bmatrix} \quad \mathbf{r}_i = \begin{bmatrix} r_{i,1} \\ \vdots \\ r_{i,T} \end{bmatrix} \quad \mathbf{t}_i = \begin{bmatrix} t_{i,1} \\ \vdots \\ t_{i,T} \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_T \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_T \end{bmatrix}$$

If f_i and g_i keep constant for T transmissions, clearly

$$\mathbf{r}_i = \sqrt{P_1 T} f_i \mathbf{s} + \mathbf{v}_i, \quad \mathbf{x} = \sum_{i=1}^{R} g_i \mathbf{t}_i + \mathbf{w}.$$

3. DISTRIBUTED SAPCE-TIME CODE

We use the idea of the linear dispersion space-time code [12] for multi-antenna systems by designing the transmitted signal at every relay node as:

$$\mathbf{t}_i = \sqrt{P_2/(P_1+1)} A_i \mathbf{r}_i \tag{1}$$

where A_i is a $T \times T$ matrix. While within the framework of linear dispersion codes, the $T \times T$ matrices A_i can be quite arbitrary (apart from a Frobenius norm constraint), to have a protocol that is equitable among different users and among different time instants, we shall henceforth assume that the A_i are unitary. This also simplifies the analysis considerably. With the normalization in (1), the average transmit power at every relay node is P_2 per transmission.

The received signal can therefore be written as,

$$\mathbf{x} = \sqrt{P_1 P_2 T/(P_1+1)} SH + W \tag{2}$$

where we have defined

$$S = \begin{bmatrix} A_1\mathbf{s} & \cdots & A_R\mathbf{s} \end{bmatrix}, \quad H = \begin{bmatrix} f_1 g_1 & \cdots f_R g_R \end{bmatrix}^t,$$

and $W = \sqrt{P_2/(P_1+1)} \sum_{i=1}^R g_i A_i \mathbf{v}_i + \mathbf{w}$.

(2) shows that the $T \times R$ matrix S works like the space-time code in multi-antenna systems. We call it the *distributed space-time code* to emphasize that it has been generated in a distributed way by the relay nodes, without having access to $\mathbf{s}$. H, which is $R \times 1$, is the equivalent channel matrix and W, which is $T \times 1$, is the equivalent noise. W is clearly influenced by the choice of the space-time code.

4. PEP AND OPTIMUM POWER ALLOCATION

When both f_i and g_i are known to the receiver, it can be calculated that $\mathbf{x}|\mathbf{s}_i$ is Gaussian with mean $\sqrt{P_1P_2T/(P_1+1)}S_iH$ and variance $\left(1 + P_2/(P_1+1)\sum_{i=1}^R |g_i|^2\right) I_T$. Therefore, $\mathrm{P}(\mathbf{x}|\mathbf{s}_i)$ is obtained and from which the ML decoding can be written as

$$\arg\min_{\mathbf{s}_i} \left\| \mathbf{x} - \sqrt{P_1P_2T/(P_1+1)} S_i H \right\|^2. \tag{3}$$

Since S_i is linear in $\mathbf{s}_i$, (3) is equivalent to the decoding of a linear system and sphere decoding can be used [15, 16].

Theorem 1 (Chernoff bound of the PEP). *With the ML decoding in (3), the PEP, averaged over the channel coefficients, of mistaking $\mathbf{s}_i$ by $\mathbf{s}_j$ has the following Chernoff bound,*

$$Pe \le \mathop{\mathrm{E}}_{g_i} \det{}^{-1} \left[I_R + \frac{P_1 P_2 T}{4\left(1 + P_1 + P_2 \sum_{i=1}^R |g_i|^2\right)} MG \right] \tag{4}$$

where $M = (S_i - S_j)^*(S_i - S_j)$ and $G = \mathrm{diag}\{|g_1|^2, \cdots, |g_R|^2\}$.

We omit the proof due to the lack of space. Let's compare (4) with the Chernoff bound on the PEP of a multi-antenna system with R transmit antennas and 1 receive antenna (The receiver knows the channel.) [4, 17]:

$$Pe \le \det{}^{-1} \left[I_R + \frac{PT}{4R} M \right].$$

The difference is that now we need to do the expectations over the g_i. Similar to the multi-antenna case, the "full diversity" condition can be obtained from (4). It is easy to see that if $S_i - S_j$ drops rank, the upper bound in (4) increases. Therefore, the Chernoff bound is minimized when $S_i - S_j$ is full-rank, or equivalently, $\det M \neq 0$.

Now let's discuss the optimum power allocation between the transmitter and relays that minimize the PEP. Because of the expectations over g_i, this is easier said than done. We shall therefore recourse to a heuristic argument. Note that $g = \sum_{i=1}^{R} |g_i|^2$ has the Gamma distribution whose mean and variance are both R. It is therefore reasonable to approximate g by its mean, i.e., $g \approx R$, especially for large R. (By the law of large numbers, almost surely $\frac{1}{R}g \to 1$ when $R \to \infty$.). Therefore, (4) becomes

$$Pe \lesssim \mathop{\mathrm{E}}_{g_i} \det{}^{-1} \left[I_T + \frac{P_1 P_2 T}{4(1+P_1+P_2 R)} MG \right]. \tag{5}$$

We can see that the upper bound in (5) is minimized when $P_1 P_2 T / 4(1+P_1+P_2 R)$ is maximized.

Assume that the total power consumed is PT for transmissions of T symbols. Since the powers used at the transmitter and every relay are P_1 and P_2 for each transmission, $P = P_1 + RP_2$. Therefore, for $P \gg 1$,

$$\frac{P_1 P_2 T}{4(1+P_1+P_2 R)} \leq \frac{P^2 T}{16R(1+P)} \approx \frac{PT}{16R} \tag{6}$$

with equality when

$$P_1 = \frac{P}{2} \quad \text{and} \quad P_2 = \frac{P}{2R}. \tag{7}$$

Therefore, the optimum power allocation is such that the transmitter uses half the total power and the relays share the other half fairly. So, for large R, the relays spend only a very small amount of power to help the transmitter.

5. APPROXIMATE DERIVATION OF THE DIVERSITY

As mentioned earlier, to obtain the diversity we need to compute the expectation in (4). We shall do this rigorously later. However, since the calculations are detailed and give little insight, we begin by giving a simple approximate derivation which leads to the same diversity result. As discussed in the previous section, when R is large, $g \approx R$ with high probability. We use this approximation in this section to simplify the derivation.

To highlight the transmit diversity result, we first upper bound the PEP using the minimum nonzero singular value of M, which is denoted as σ^2_{min}. From (4) and (6),

$$\begin{aligned} Pe &\lesssim \mathop{\mathrm{E}}_{g_i} \det{}^{-1} \left[I_T + \frac{PT\sigma^2_{min}}{16R} \mathrm{diag}\,\{I_{\mathrm{rank}\,M}, 0\} G \right] \\ &= \mathop{\mathrm{E}}_{g_i} \prod_{i=1}^{\mathrm{rank}\,M} \left(1 + \frac{PT\sigma^2_{min}}{16R} |g_i|^2 \right)^{-1} \\ &= \left[\int_0^\infty \left(1 + \frac{PT\sigma^2_{min}}{16R} x \right)^{-1} e^{-x} dx \right]^{\mathrm{rank}\,M} \\ &= \left(\frac{PT\sigma^2_{min}}{16R} \right)^{-\mathrm{rank}\,M} \left[-e^{-\frac{16R}{PT\sigma^2_{min}}} \mathbf{Ei} \left(-\frac{16R}{PT\sigma^2_{min}} \right) \right]^{\mathrm{rank}\,M} \end{aligned}$$

where $\mathbf{Ei}(\chi) = \int_{-\infty}^{\chi} \frac{e^t}{t} dt,\ \ \chi < 0$ is the exponential integral function [18]. Also, $\mathbf{Ei}(\chi) = c + \log(-\chi) + \sum_{k=1}^{\infty} \frac{(-1)^k \chi^k}{k \cdot k!}$ with c the Euler constant. For $P \gg 1$, $e^{-\frac{16R}{PT\sigma^2_{min}}} \approx 1$ and $-\mathbf{Ei}\left(-\frac{16R}{PT\sigma^2_{min}} \right) \approx \log P$. Therefore,

$$\begin{aligned} Pe &\lesssim \left(16R/T\sigma^2_{min}\right)^{\mathrm{rank}\,M} \left(\log P/P\right)^{\mathrm{rank}\,M} \\ &= \left(16R/T\sigma^2_{min}\right)^{\mathrm{rank}\,M} P^{\mathrm{rank}\,M\left(1-\frac{\log\log P}{\log P}\right)} \end{aligned} \tag{8}$$

When M is full rank, the transmit diversity is $\min\{T, R\} \left(1 - \frac{\log\log P}{\log P}\right)$. Therefore, similar to the multi-antenna case, there is no point in having more relays than the coherence interval. Thus, we will henceforth assume $T \geq R$. The transmit diversity is therefore $R\left(1 - \frac{\log\log P}{\log P}\right)$. (8) also shows that the PEP is smaller for bigger coherence interval T. A tighter upper bound is given in the following theorem.

Theorem 2. *Design the transmit signal at the ith relay node as in (1) and use the power allocation in (7). If $P \gg \log P$, the PEP has the following upper bound*

$$Pe \lesssim \sum_{k=0}^{R} \left(\frac{16R}{T} \right)^k \sum_{1 \leq i_1 < \cdots < i_k \leq R} \det{}^{-1} [M]_{i_1,\cdots,i_k} \frac{\log^k P}{P^R} \tag{9}$$

where $[M]_{i_1,\cdots,i_k}$ denotes the $k \times k$ matrix composed by choosing the $i_1, \cdots, i_k$-th rows and columns of M.

Idea of the proof: To upper bound the R integrals in (4), we first break every integral into two parts: the integration from 0 to $1/P$ and the integration from $1/P$ to ∞, and then upper bound every one of the resulting 2^R terms. □

The $k = R$ term in (9) has the highest order of P.

$$\det{}^{-1} M \left(16R/T\right)^R \left(\log P/P\right)^R \tag{10}$$

Therefore, as in (8), the transmit diversity of the distributed space-time code is, again, $R\left(1 - \frac{\log\log P}{\log P}\right)$, which is linear in the number of relays. When P is very large ($P \gg \log P$), $\frac{\log\log P}{\log P} \ll 1$, and a transmit diversity about R is obtained which is the same as the transmit diversity of a multi-antenna system with R transmit antennas. That is, the system works as if the R relay nodes fully cooperate and have full knowledge of the transmitted signal. Generally, the transmit diversity depends on the transmit power P.

6. RIGOROUS DERIVATION OF THE DIVERSITY

In Section 5, we used the approximation $\sum_{i=1}^{R} |g_i|^2 \approx R$. In this section, a rigorous derivation of the Chernoff upper bound on the PEP is given. The same transmit diversity is obtained but the coding gain becomes more complicated. In fact, this bound is tighter. Here is the main result.

Theorem 3. *Design the transmit signal at the ith relay node as in (1) and use the power allocation in (7). If $P \gg \log P$, the PEP Chernoff bound is given in (11). When $R \gg 1$,*

$$Pe \lesssim \sum_{k=0}^{R} \left(\frac{8R}{T} \right)^k \sum_{1 \leq i_1 < \cdots < i_k \leq R} \det{}^{-1} [M]_{i_1,\cdots,i_k} \frac{\log^k P}{P^R}. \tag{12}$$

$$Pe \lesssim \sum_{k=0}^{R} \left(\frac{8}{T}\right)^k \sum_{1\leq i_1<\cdots<i_k\leq R} \sum_{l=0}^{k} B_R(k-l,k) \det{}^{-1}[M]_{i_1,\cdots,i_k} \frac{\log^l P}{P^R}$$
$$B_R(j,k) = \begin{pmatrix} k \\ j \end{pmatrix} \sum_{i_1=1}^{k} \sum_{i_2=1}^{k-i_1} \cdots \sum_{i_j=1}^{k-i_1-\cdots-i_{j-1}} \begin{pmatrix} k \\ i_1 \end{pmatrix} \cdots \begin{pmatrix} k-i_1-\cdots-i_{j-1} \\ i_j \end{pmatrix} (i_1-1)!\cdots(i_j-1)! R^{k-i_1-\cdots-i_j} \quad (11)$$

Remarks:

1. The term with the highest order of P in (11) is:

$$\det{}^{-1} M \,(8R \log P/TP)^R \quad (13)$$

which is the same as (10) except for a coefficient of 2^R. Therefore, the same transmit diversity is obtained.

2. (12) also gives the coding gain for very large P ($P \gg \log P$). The dominant term in (12) is (13). The coding gain is therefore $\det M$, which is the same as the multi-antenna case. When P is not very large, the $k = R-1$ term cannot be ignored and even the $k = R-2, \cdots$ terms have non-neglectable contributions. Therefore, we want not only $\det M$ to be large but also $\det[M]_{i_1,\cdots,i_k}$ to be large for all $0 \leq k \leq R, 1 \leq i_1 < \cdots < i_k \leq R$. Note that

$$[M]_{i_1,\cdots,i_k} = ([S_i]_{i_1,\cdots,i_k} - [S_j]_{i_1,\cdots,i_k})^*([S_i]_{i_1,\cdots,i_k} - [S_j]_{i_1,\cdots,i_k})$$

where $[S_i]_{i_1,\cdots,i_k} = (A_{i_1}s_i, \cdots, A_{i_k}s_i)$ is the space-time code when only the $i_1, \cdots, i_k$th relay nodes are working. To have good performance for general P, the distributed space-time code should be "scale-free" in the sense that it is still a good distributed space-time code when some of the relays are not working. In general, for networks with any number of relay nodes, the same conclusion can be obtained from (11).

3. If $P \ll 1$, with the approximation $\sum_{i=1}^{R} |g_i|^2 \approx R$ and the power allocation given in (7), (4) can be calculated to be

$$Pe \lesssim 1 - \frac{P^2 T}{16R} \operatorname{tr} M + o(P^2)$$

The same as the multi-antenna case, the coding gain is $\operatorname{tr} M$. The design criterion is to maximize $\operatorname{tr} M$.

4. Theorem 3 also shows that the rigorous derivation in this section is consistent with the approximate derivation except for a coefficient 2^k. Actually the upper bound in (12) is tighter than the one in (9). This is because in (12) all the terms except the one with the highest order of R are omitted, however in the derivation of (9), we approximate g by its mean.

7. SIMULATION RESULTS

We compare the performance of LD codes implemented distributively over wireless networks with that of the same codes in multi-antenna systems. In this paper, the actually design of the LD code and their optimality is not an issue. Therefore, we generate the matrices A_i randomly based on the isotropic distribution. The transmit signals at relays are designed as in (1). s_t are designed as independent N^2-QAM signals. Therefore, the rate of the code is $2 \log N$.

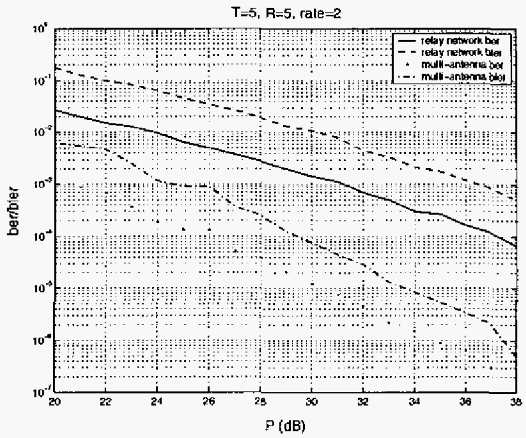

Fig. 1. Relay network vs. multi-antenna system.

In the first example, $T = R = 5$ and $N = 2$. The rate is 2 bits per transmission. Fig. 1 shows that the performance of the multi-antenna system is always better than the relay network at any P. This is what we expected because in the multi-antenna system, the transmit antennas can fully co-operate and have perfect information of the transmit signal. Also we can see from Fig. 1 that the ber and bler curves of the multi-antenna system goes done faster than those of the relay network. However, the differences of the slopes are diminishing as the total transmit power goes higher. This verifies our analysis of the transmit diversity.

In the second example, $T = R = 20$ and $N = 2$. The rate is again 2. From Fig. 2 we can see that the ber/bler curve of the multi-antenna system decreases faster than the ber/bler of the relay network. However, as P goes high ($P > 20$), they are about the same slope.

8. CONCLUSION

We propose the use of linear dispersion space-time codes in a wireless relay network. We assume that the transmitter and relay nodes do not know the channel realizations but only their statistical distribution. The ML decoding and PEP at the receiver is analyzed. The main result is that the diver-

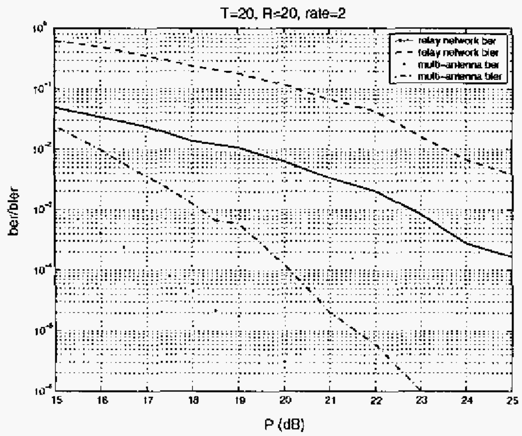

Fig. 2. Relay network vs. multi-antenna system.

sity of the system behaves as $\min\{T, R\}\left(1 - \frac{\log\log P}{\log P}\right)$, which shows that when $T \geq R$ and the average total transmit power is very high ($P \gg \log P$), the relay network has almost the same diversity as a multi-antenna system with R transmit antennas. This result is also supported by simulations. We also observe that the high SNR coding gain, $|\det(S_i - S_j)|^{-2}$, is the same as what arises in space-time coding. The same is true at low SNR where a trace condition comes up.

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