On the power efficiency of Sensory and Ad-hoc wireless networks[1]

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Abstract — **We consider the power efficiency of a communications channel, i.e., the maximum bit rate that can be achieved per unit power (energy rate). For AWGN channels, it is well known that power efficiency is attained in the low SNR regime. In this paper we show that for a random sensory, or ad-hoc, wireless network with n users (nodes), with high probability converging to one as n grows, the power efficiency scales at least by a factor of $\sqrt{n}$. In other words, each user in a wireless channel with n nodes can support the same communications rate as a single user system, but by expending only $\frac{1}{\sqrt{n}}$ the energy.**

I. Introduction

We consider the power efficiency, i.e., the maximum ratio between total bit rate and total power, of sensory and ad-hoc wireless networks. We assume that in both cases, n users are located randomly on a domain of fixed area. At each time there are r receive/transmit pairs and all the other nodes are considered as relay nodes(for the sensory case $r = 1$.) The results on the capacity of sensory and ad-hoc wireless networks, (e.g. [1], [2]) show that the per-user capacity scales as $\Theta(\frac{\log n}{n})$ and $\Theta(\frac{\sqrt{n}}{n})$, respectively. In either case, the results are discouraging from a practical point of view since they represent rapidly diminishing to zero rewards as the number of nodes (users) in the network increases. In this paper we show that by operating the network at low SNR and by exploiting, rather than avoiding, the interference inherent in any wireless network we are able to achieve a power efficiency that scales as the size of the network grows. Also we propose a so-called "Listen and Transmit Protocol" for achieving this power efficiency. The channel model used in this paper is quite general. The channel between two points x and y is a random variable, denoted by c_{xy}. We assume that, averaged over the fading, the various channels c_{xy} are independent. For our result on sensory networks this suffices. For ad-hoc networks, we will further assume that for any fixed points y, z, t and w

$$\begin{aligned} \mathrm{E}_{\{x\}}\mathrm{E}\, c_{xy} &= 0 \\ \mathrm{E}_{\{x\}}\mathrm{E}\, c_{xy}c_{xz} &= 0 \\ \mathrm{E}_{\{x\}}\mathrm{E}\, c_{xy}c_{xz}c_{xt}c_{xw} &= 0 \end{aligned} \tag{1}$$

where $\mathrm{E}_{\{x\}}$ denotes the average over the location of point x and the inner expectation is over the fading of the channels. The above condition is clearly met if the fading is zero-mean. We denote the transmit node and relay node powers by p and σ_r^2 respectively.

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II. Power Efficiency Results

Theorem 1 *Consider an n node random sensory network in a domain of fixed area where averaged over the fading, the various channels are independent. Assume that the measurement noises are all iid zero mean complex Gaussian random variables. Then with high probability the power efficiency of the network, η, is at least $\Theta(\sqrt{n})$, i.e.,*

$$\Pr\{\eta > K_1\sqrt{n})\} \geq 1 - \frac{K}{n} \tag{2}$$

where K_1 and K are independent of n but depend on the domain and the fading characteristics. Moreover, the listen transmit protocol achieves $\eta = \Theta(\sqrt{n})$ with the choice of $p = n\sigma_r^2 = O(\frac{1}{\sqrt{n}})$.

Theorem 2 *Consider an n node random ad-hoc network with the assumptions used in Theorem 1. Furthermore assume (1) and that at any given time there are $r = O(n^\nu)$ $\nu \leq \frac{1}{2}$ transmit/receive pairs. If we denote the power efficiency of the network by η, then for every $\mu > 0$*

$$\Pr\{\eta > K_1 n^{1-\max\{\frac{1}{2},\frac{\mu+2\nu}{2}\}}\} \geq 1 - \frac{K_2}{n^{\min\{1-2\nu,\mu\}}} \tag{3}$$

where K_1 and K_2 are independent of n and r but depend on the domain and the fading characteristics Moreover, the listen and transmit protocol achieves this lower bound.

The following corollary follows immediately.

Corollary 1 *Assume the conditions of Theorem 2. If the number of transmit/receive pairs in the network is of $O(n^{\frac{1-\epsilon}{2}})$, where $\epsilon > 0$, then we have*

$$\Pr\{\eta > K_3\sqrt{n}\} \geq 1 - \frac{K_4}{n^\epsilon} \tag{4}$$

where K_3 and K_4 are independent of n and r but depend on the domain and the fading characteristics. Moreover, by choosing the transmit and relay node power as $rp = n\sigma_r^2 = \Theta(\frac{r}{\sqrt{n}})$, the listen and transmit protocol achieves this lower bound.

References

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