

# Blind Identification of FIR Channels via Antenna Arrays

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## Abstract

We propose a new method for blind identification of possibly nonminimum phase FIR channels using antenna arrays. Based on a frequency domain formulation, we show that even a nonminimum phase channel driven by stationary sequences can be identified from the second-order statistics. Although in the single antenna case it is necessary to use cyclostationary signals or higher order statistics to identify the magnitude and phase of the channel, we circumvent such a requirement by exploiting multichannel properties of the array. We also present necessary and sufficient conditions for channel identifiability, and develop an identification algorithm.

## 1 Introduction

The need for blind channel equalization is growing in digital communications systems. Equalization of a communications channel requires implicit or explicit knowledge of its transfer function. A communication channel is usually identified by LMS or recursive least-squares (RLS) type adaptive algorithms where the reference signal is provided by transmitting known training sequences. The so-called blind channel identification techniques only use the channel output and some known statistical properties of the transmitted signal to identify the channel. As a result, these techniques have the potential to increase the transmission capability of the communication channel by eliminating training sequences.

It is well known that nonminimum phase channels driven by wide-sense stationary input sequences, cannot be identified from second-order statistics. There-

fore blind identification techniques, to date, use either higher order statistics [1, 2, 3, 4, 5, 6] or exploit cyclostationarity of input signals [7, 8, 9] to identify a possibly nonminimum phase channel. The main drawback of the former techniques is their slow rate of convergence which makes them impractical for mobile communications environments with sudden fades and channel changes. The idea of exploiting cyclostationary properties of communication signals has been mainly pursued by Gardner and applications to channel identification were also presented [7].

Recently, L. Tong *et al* [10] have proposed a new method to identify FIR channels using a fractionally spaced equalization structure that exploits the cyclostationarity of the input sequence.

In general, signals arrive at the receiver not only with different delays, but also from different spatial angles. The spatial properties of the multipath channel cannot be exploited with a single antenna receiver, and it is necessary to use antenna arrays to achieve a more efficient detection scheme. In digital transmission systems, antenna arrays have recently attracted much attention in the framework of optimal spatial diversity combining [11]. In addition to improving the performance in low SNR environments, an antenna array greatly reduces the effect of interference sources by exploiting the spatial structure of the multipath channel and directing nulls towards undesired users. While the popular techniques for angle of arrival (AOA) estimation are the so-called subspace techniques (MUSIC and ESPRIT), these methods are usually inapplicable to communications environments since the number of multipaths typically exceeds the number of sensors. Also in situations where the relative delays of different paths are much smaller than the bit interval, the coherency of the received signals from such paths causes difficulties for these methods.

In this paper, we present a method for identifying possibly nonminimum phase FIR channels via antenna arrays that is based on the computation of the second-

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order statistics of the received signal. Tong *et al* [12] have recently shown that spatial diversities of multiple receivers improves the performance of the blind identification schemes for cyclostationary signals. As we show in this paper, for the antenna array case, second-order statistics provide enough information for identifying the channel and it is not necessary to use higher order statistics or cyclostationary signals. The proposed method circumvents the necessity of the above mentioned requirements by exploiting the multichannel property of the antenna array.

The algorithm is based on a frequency domain approach that forms the spectrum matrix  $S_{\mathbf{x}}(z)$  of the received signal  $\mathbf{x}(t)$ .  $S_{\mathbf{x}}(z)$  can be easily obtained by computing the autocorrelation function  $R_{\mathbf{x}}(\tau)$  of the input signal at different time delays and writing it down in the  $z$ -domain. The impulse response of the different elements of the channel can then be estimated by computing the common factors of each row of  $S_{\mathbf{x}}(z)$ .

In applications where the environment can be represented by a discrete multipath model, the delay and direction of arrival (DOA) vector of each path can be estimated in the proposed framework. In addition, as long as different paths are well separated in time such that at each sampling point of the channel impulse response only one path is present, the angles of arrivals of the paths can also be estimated. In these scenarios, DOA angles can be estimated even in situations where the total number of paths is more than the number of antenna elements. An example for this scenario is given in Sec. 3 of this paper.

## 2 Problem Formulation

### 2.1 Data Model

In a PAM data communication system the baseband transmitted signal  $u(t)$  is given by

$$u(t) = \sum_{k=-\infty}^{\infty} s_k p(t - kT)$$

where  $T$  is the symbol period;  $p(\cdot)$  is the pulse shaping function, and  $s_k$  is the transmitted sequence.

The signal received at the  $i$ th antenna element is then given by

$$x_i(t) = h_i^c(t) \otimes u(t) + n_i(t)$$

where  $h_i^c(\cdot)$  is the channel impulse response of the  $i$ th antenna, and  $n_i(\cdot)$  is the corresponding additive noise

process. By combining pulse shaping and channel impulse response functions in a composite channel response  $h_i(t)$ , we will get

$$x_i(t) = \sum_{k=-\infty}^{\infty} s_k h_i(t - kT) + n_i(t)$$

and the output of an antenna array with  $M$  elements will be of the form

$$\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T$$

In our data model we will make the following assumptions:

1. The channel impulse responses  $h_i(t)$  are assumed to be finite impulse response (FIR) filters.
2. Noise terms  $n_i(t)$  are zero-mean, uncorrelated with  $s_k$ , and white in time and space

$$E[n_i(t_1)n_j(t_2)] = \begin{cases} \sigma^2 & \text{when } i = j \text{ and } t_1 = t_2 \\ 0 & \text{otherwise} \end{cases}$$

3. For convenience, the transmitted sequence  $s_k$  is assumed to be zero-mean and iid,  $E[s_k s_l] = \delta(k - l)$ , although the algorithm still works for a general input with known power spectral density.

### 2.2 Multichannel Identification Approach

The proposed approach is based on computation of the second order statistics of the received signal. Denote the autocorrelation of  $\mathbf{x}(t)$  by

$$R_{\mathbf{x}}(t, \tau) = E[\mathbf{x}(t)\mathbf{x}^*(t - \tau)]$$

By sampling the output signal  $\mathbf{x}(t)$  at symbol rate  $1/T$ , the discrete signal

$$\mathbf{x}[k] = \mathbf{x}(kT)$$

will be a wide-sense stationary signal with autocorrelation function

$$R_{\mathbf{x}}[m] = E[\mathbf{x}[k]\mathbf{x}^*[k - m]]$$

With  $h_i[k] = h_i(kT)$ , the elements of  $R_{\mathbf{x}}[m]$  can be written as

$$\begin{aligned} R_{x_i, x_j}[m] &= E[x_i[k]x_j^*[k - m]] \\ &= \sum_l \sum_{l'} E(s_l s_{l'}) h_i[k - l] h_j^*[k - m - l'] \\ &\quad + \sigma^2 \delta[m] \delta[i - j] \\ &= E(s_i^2) h_i[m] \otimes h_j^*[-m] + \sigma^2 \delta[m] \delta[i - j] \end{aligned}$$

The power spectrum of the output array,  $S_{\mathbf{x}}(z)$ , defined to be the  $z$ -transform of  $R_{\mathbf{x}}[m]$ , is then given by

$$S_{\mathbf{x}}(z) = Z(R_{\mathbf{x}}[m]) = \begin{pmatrix} H_1(z)H_1^*(z^{-1}) & \dots & H_1(z)H_M^*(z^{-1}) \\ H_2(z)H_1^*(z^{-1}) & \dots & H_2(z)H_M^*(z^{-1}) \\ \vdots & \ddots & \vdots \\ H_M(z)H_1^*(z^{-1}) & \dots & H_M(z)H_M^*(z^{-1}) \end{pmatrix} + \sigma^2 I \quad (1)$$

which can be written as

$$S_{\mathbf{x}}(z) = \mathbf{H}(z)\mathbf{H}^*(z^{-1}) + \sigma^2 I \quad (2)$$

where

$$\mathbf{H}(z) = [H_1(z), H_2(z), \dots, H_M(z)]^T$$

is the multichannel transfer function vector.

It is interesting to note that although  $R_{\mathbf{x}}(\tau)$  has full rank, in the noise free case  $S_{\mathbf{x}}(z)$  will have rank one. This is independent of the number of multipath present, and will be crucial for identifying the channels.

For the single antenna case, it is not possible to distinguish between a minimum-phase and nonminimum-phase channel, due to the fact that only the  $(1, 1)$  element  $H_1(z)H_1^*(z^{-1})$  in (1), can be obtained from the second-order statistics. However, as can be easily verified from (1), in the multiple antenna situation, the cross-correlation between different antennas provides valuable phase information that can be used to identify all the channels. For example, in the noiseless situation,  $H_i(z)$ , can be simply estimated by computing the common factor of the elements of  $i$ th row of  $S_{\mathbf{x}}(z)$ . White noise processes can also be handled; either by estimating the noise covariance  $\sigma$ , or by neglecting the diagonal terms of  $S_{\mathbf{x}}(z)$  and computing the common factor of the remaining terms in each row.

In general, a channel is called identifiable if it can be determined up to a scalar constant factor. In this respect a channel will be identifiable from (1) if, and only if, the common factor of each row is identical to the corresponding channel transfer function. Thus, the following result is expected.

**Multichannel Identifiability Condition** *The  $M$  channels  $h_i[n]$  are identifiable up to a constant factor if, and only if, there are no common roots between all of the channel transfer functions  $\{H_i(z)\}_{i=1}^M$ .*

In fact if all the channels have a common root at  $z = \alpha$ , then the term  $(z - \alpha)(z^{-1} - \alpha^{-*})$  can be factored out of

$S_{\mathbf{x}}(z)$ , and it is not possible to resolve the ambiguity of this root. However, we should note that any  $M$  randomly chosen polynomials will *generically* share no common roots, and in particular as  $M$  increases it is less likely that they shall share a common root.

Although the common factor of a number of polynomials can be computed using standard techniques such as Euclid's algorithm, such approaches are sensitive to noise and finite data imperfections. In what follows, we shall present a more robust method that is based upon computing the smallest singular vector of a certain generalized Sylvester resultant matrix.

## 2.3 Channel Identification Algorithm

In this section we shall present a method for identifying the channels  $\{H_i(z)\}_{i=1}^M$  from the power spectrum matrix (1). We shall assume that either we know (or have estimated) the noise covariance  $\sigma^2 I$ , and have subtracted it out of  $S_{\mathbf{x}}(z)$ , or we considered its off-diagonal elements. Consider the  $i$ th and  $j$ th rows of the power spectrum:

$$S_{x_i x_k}(z) = H_i(z)H_k^*(z^{-1}) = \sum_{l=-L}^L S_{x_i x_k}^{(l)} z^l$$

$$S_{x_j x_k}(z) = H_j(z)H_k^*(z^{-1}) = \sum_{l=-L}^L S_{x_j x_k}^{(l)} z^l$$

for  $k = 1, 2, \dots, M$ , and where  $L$  is the length of the channel. Note that

$$S_{x_j x_k}(z)H_i(z) - S_{x_i x_k}(z)H_j(z) = 0 \quad k = 1, 2, \dots, M$$

By equating the coefficients of the powers of  $z$  in the above equation we have

$$\mathcal{S}(S_{x_j x_k}, -S_{x_i x_k}) \begin{bmatrix} h_{i0} \\ h_{i1} \\ \vdots \\ h_{iL} \\ h_{j0} \\ h_{j1} \\ \vdots \\ h_{jL} \end{bmatrix} = 0 \quad k = 1, 2, \dots, M \quad (3)$$

where  $H_i(z) = \sum_{l=0}^L h_{il}z^l$ ,  $H_j(z) = \sum_{l=0}^L h_{jl}z^l$  and  $\mathcal{S}(a, b)$  is the  $(3L + 1) \times (2L + 2)$  Sylvester matrix defined as:

$$S(a, b) = \begin{bmatrix} a_0 & & & b_0 & & & \\ a_1 & a_0 & & b_1 & b_0 & & \\ \vdots & a_1 & \ddots & \vdots & b_1 & \ddots & \\ a_{2L} & \vdots & \ddots & a_0 & b_{2L} & \vdots & b_0 \\ & a_{2L} & & a_1 & b_{2L} & b_{2L} & b_1 \\ & & \ddots & \vdots & & \ddots & \vdots \\ & & & a_{2L} & & & b_{2L} \end{bmatrix}$$

with  $a(z) = \sum_{l=-L}^L a_{L+l}z^l$ ,  $b(z) = \sum_{l=-L}^L b_{L+l}z^l$ . Equation (3) can be rewritten as:

$$\begin{bmatrix} S(S_{x_j x_1}, -S_{x_i x_1}) \\ \vdots \\ S(S_{x_j x_M}, -S_{x_i x_M}) \end{bmatrix} \begin{bmatrix} h_{i0} \\ \vdots \\ h_{iL} \\ h_{j0} \\ \vdots \\ h_{jL} \end{bmatrix} = S_{i,j} \begin{bmatrix} h_{i0} \\ \vdots \\ h_{iL} \\ h_{j0} \\ \vdots \\ h_{jL} \end{bmatrix} = 0 \quad (4)$$

We now have the following result:

**Theorem** The channels  $\{H_i(z)\}_{i=1}^M$  are uniquely identifiable if, and only if,  $S_{i,j}$  has a nullspace of dimension one. In this case the coefficients of the  $i$ th and  $j$ th channels are given by solving (4).

This Theorem is of particular interest since it relates channel identifiability to the power spectrum that we can estimate, and because it motivates the following algorithm for channel identification. It should be noted that it is also possible to combine Sylvester matrices corresponding to more than 2 channels. In this way the performance of the estimated channels can be improved in the expense of more complexity.

In practice we can only estimate the power spectrum from finite samples. Hence

$$\hat{S}_{x_j x_k}(z)H_i(z) - \hat{S}_{x_i x_k}(z)H_j(z) \neq 0 \quad k = 1, 2, \dots, M$$

where  $\hat{S}$  denotes estimates of the power spectrum. We propose the following criterion to estimate the channels:

$$\min_{H_i(z), H_j(z) \neq 0} \sum_{k=0}^M \|\hat{S}_{x_j x_k}(z)H_i(z) - \hat{S}_{x_i x_k}(z)H_j(z)\|_2^2$$

The solution is given by the smallest right singular vector of  $\hat{S}_{i,j}$ . We thus have the following algorithm:

#### Blind Channel Identification Algorithm

- Estimate the power spectrum  $S_x(z)$  to obtain  $\hat{S}_x(z)$ .
- Form  $\hat{S}_{i,j}$ .
- Calculate the smallest right singular vector  $\hat{S}_{i,j}$  to obtain estimates of the coefficients of  $H_i(z)$  and  $H_j(z)$ .

Note that in this method we have only used the  $i$ th and  $j$ th rows of the spectrum to compute  $H_i(z)$  and  $H_j(z)$ . The remaining rows of  $S_x(z)$  may also be used in a similar fashion.

### 3 Simulation Results

In this section we study the performance of our channel identification algorithm. We consider a case where the signal is received from 8 different paths, and where the attenuations and angles of arrival of each path are shown in Figures 1 and 2 respectively. In this example we have used 3 antenna elements and an SNR of 20db. These 8 paths induce 3 channels in the 3 antenna elements, and we shall use our proposed method to estimate all 3 channels at the same time.

It should be noted that in this example we have chosen different paths to be well separated in time, at least by one bit interval. This results in 3 channels of length  $L=7$  such that the angles of arrival may be obtained from the coefficients of the channels' impulse responses. If different paths were not separated in time in such a manner, it would still be possible to estimate the channels, however, it may not be possible to obtain the DOAs from the channel impulse response coefficients.

The results of Figures 1 and 2 are given for 100 independent Monte Carlo runs using 500 data samples of a BPSK signal. The results show that although the number of paths is much larger than the number of antenna elements, the angles of arrival can be estimated quite reasonably by exploiting the structure of the input signal. Figure 1 shows the mean and standard deviation of the estimated channel impulse response for the first antenna element, normalized with respect to the main path. In Figure 2, the performance of estimated angles is shown in the same scenario.

### 4 Concluding Remarks

A new method for blind identification of FIR channels via antenna arrays is proposed. By exploiting the multichannel structure of the system, the proposed method can identify a possibly non-minimum phase channel using second-order statistics of the input signal. To conclude the paper we shall mention some possible extensions of the above mentioned algorithm.

- The channel identification algorithm only requires the calculation of the smallest right singular vector of  $\hat{S}_{i,j}$ . Since  $\hat{S}_{i,j}$  is a highly structured matrix, this structure may be exploited to obtain fast adaptive identification schemes.

- If cyclostationary signals are present, one may use the cyclic spectrum  $S_{\mathbf{x}}^{\alpha}(z)$  instead of  $S_{\mathbf{x}}(z)$ . The arguments are easily generalized.
- Oversampling may be used to increase the resolution of the estimated transfer functions in time domain or to improve the rate of convergence of estimating the second order statistics of the received signal.
- If we have more than one independent source present, it is easy to see that the rank of  $S_{\mathbf{x}}(z)$  in the noiseless case will be equal to the number of independent sources (provided this number is less than  $M$ , the number of antenna elements). Extension of the present formalism to the multi-user case is currently under investigation.

Figures

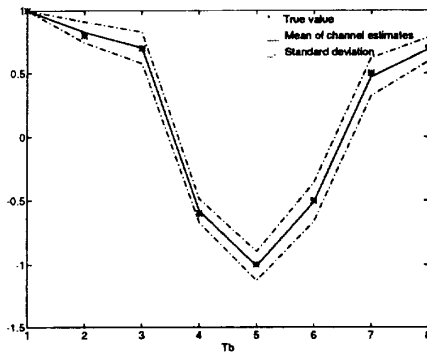


Figure 1: Mean and standard deviation of 100 estimates of channel impulse response using  $M=3$  antennas, and 500 samples. SNR=20db

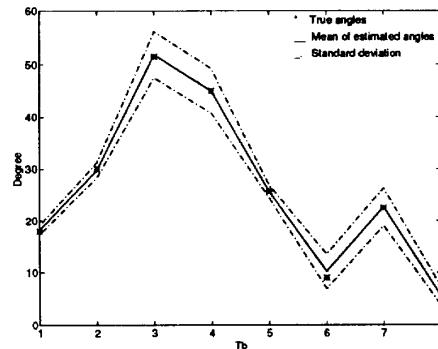


Figure 2: Mean and standard deviation of 100 estimates of angles of arrivals of each path using  $M=3$  antennas, and 500 samples. SNR=20db

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