On the Power Efficiency of Sensory and Ad-Hoc Wireless Networks

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Wireless Networks

- wireless is a *shared* medium
  - network is a fully-connected graph
  - bandwidth is precious
- distinguishing features
  - interference
  - path-loss
  - fading

Traditional methods attempt to combat these "deficiencies".

A better approach is to *exploit* them.

Sensory Networks and Ad-Hoc Networks

Sensory Networks:

- applications: environmental, surveillance, etc.
- all information intended for a single receiver
- sensors may only need to occasionally transmit information

Ad-Hoc Networks:

- no infrastructure
- users "somehow" cooperate in a distributed fashion to communicate
- can have many users acting simultaneously as transmitters or receivers

Some Scaling Laws

How does the capacity of such networks scale in n, the number of nodes?

- for sensory networks capacity scales as $O(\log n)$ (Gastpar and Vetterli, 2002)
- for ad-hoc networks capacity scales *at least* as $O(\sqrt{n})$ (Gupta and Kumar, 2000)

Both are overall discouraging results:

- in sensory networks, the capacity per participating node is $O(\frac{\log n}{n})$
- in ad-hoc networks, the capacity per participating node is $O(\frac{1}{\sqrt{n}})$

In either case, these are diminishing returns for the nodes' energy investment and participation in the communications

Power Efficiency

- in wireless networks, especially sensory networks, power consumption is a major bottleneck
- this had led many researchers[a] to study bits/energy

Following Verdu (2002), we shall rather look at (bits/sec)/(energy/sec), i.e., at *power efficiency* defined as

$$\eta = \frac{\text{capacity}}{\text{transmit power}}$$

For an AWGN channel this is

$$\eta = \frac{\log(1 + \sigma_s^2/\sigma_n^2)}{\sigma_s^2}$$

[a] Ephremides et al, El Gamal et al, Goldsmith et al, etc.

Power Efficiency

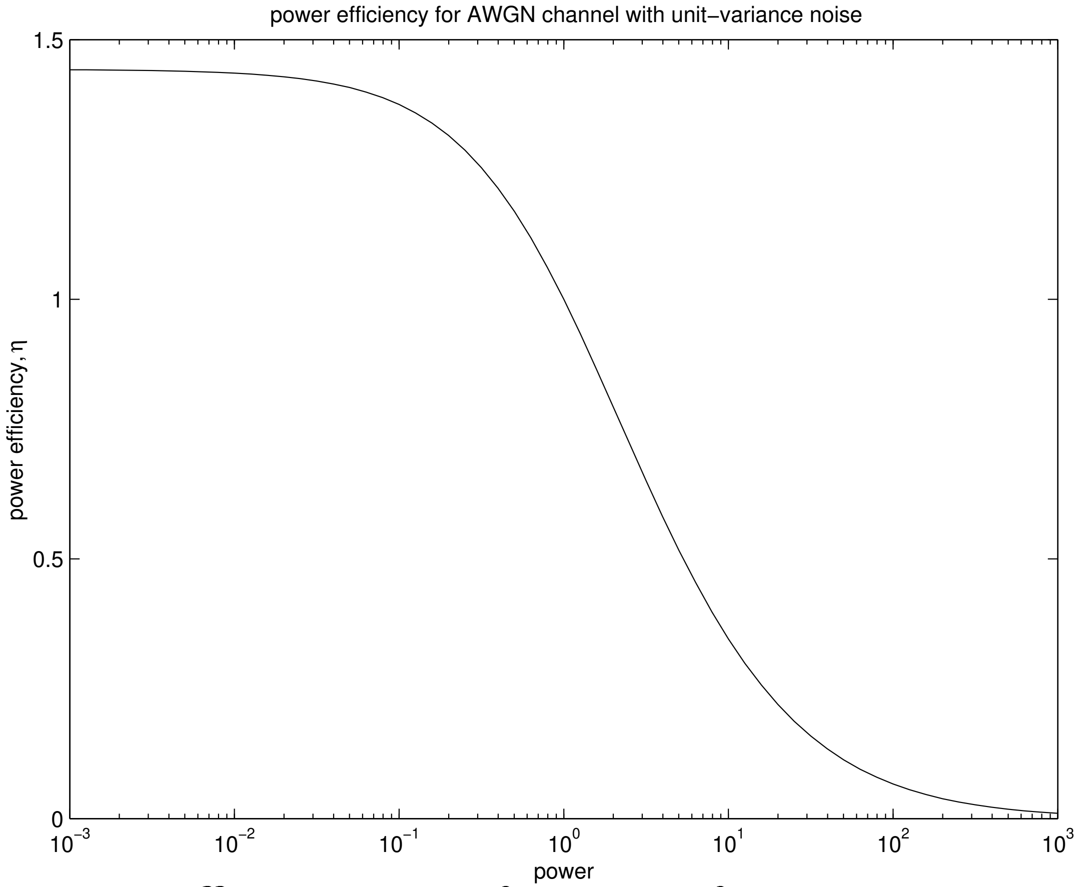


Figure 1: Power efficiency as a function of transmit power for an AWGN channel with unit-variance noise

Clearly,

- as $\sigma_s^2 \to \infty$:
$$\eta \to \frac{\log \sigma_s^2}{\sigma_s^2} \to 0,$$
which means we are power in-efficient at high SNR
- as $\sigma_s^2 \to 0$:
$$\eta = \frac{\log(1 + \sigma_s^2/\sigma_n^2)}{\sigma_s^2} \to \frac{\log e}{\sigma_n^2} = O(1),$$
which means we are power efficient at low SNR

This was all from a single-user perspective.

What happens in a network setting?...

Multi-Antenna Power Efficiency

To gain some insight, consider an n-transmit singel-receive multi-antenna channel, with channel matrix:

$$H = \left[\begin{array}{cccc} h_1 & h_2 & \ldots & h_n \end{array} \right] \quad , \quad E|h_i|^2 = 1$$

- if the channel matrix is *known* to the transmitter (beamforming):

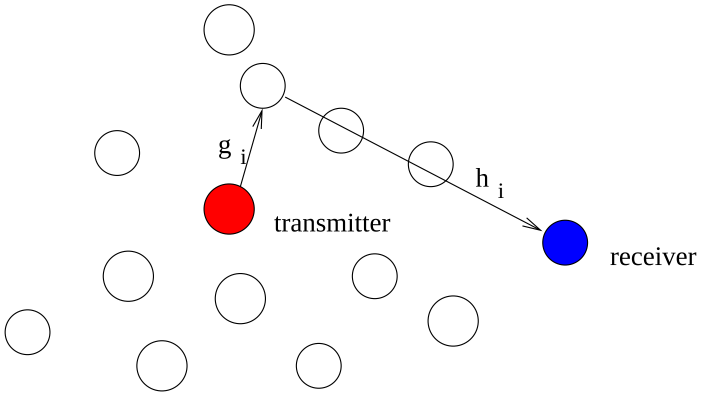
$$\eta = \frac{E \log\left(1 + \frac{\sigma_s^2}{\sigma_n^2}(\sum_{i=1}^n |h_i|^2)^2\right)}{n\sigma_s^2} \to \frac{n \log e}{\sigma_n^2} = O(n)$$

- if the channel matrix *unknown* to the transmitter (Foschini-Telatar):

$$\eta = \frac{E \log\left(1 + \frac{\sigma_s^2}{\sigma_n^2} \sum_{i=1}^n |h_i|^2\right)}{n\sigma_s^2} \to \frac{\log e}{\sigma_n^2} = O(1)$$

From a power efficiency point of view, it only pays off to have multiple transmit antennas if the channel is known at the transmitter.

Sensory Networks



- gains from transmitter to each node: $g_i, \quad i = 1, \ldots, n$
- gains from each node to receiver: $h_i, \quad i = 1, \ldots, n$
- **Assumptions:**
  - $\{g_i, h_i\}$ are independent, zero-mean, unit-variance with finite fourth-order moment
  - system is synchronous
  - each node i knows its local connections, g_i and h_i, but not the rest of the network

Remarks on the assumptions;

- having the $\{g_i, h_i\}$ independent, with finite fourth-order moment is very reasonable
- zero-mean condition can be relaxed
- synchronization and local knowledge of the $\{g_i, h_i\}$ also reasonable
- only questionable assumption is the unit-variance assumption since it ignores the geometry of the network. However,
  - if n is large and we have *spatial ergodicity* it is a reasonable assumption
- Results are robust, since they only depend on the first- and second-order statistics of the channel gains

A "Listen and Transmit" Protocol

Communication is divided into two intervals:

1. The *listen* interval: the transmitter transmits the signal s, with power p, and all other nodes are silent, but listen:

$$q_i = g_i s + v_i, \quad i = 1, \ldots, n$$

2. The *transmit* interval: each node transmits with power σ_s^2 the signal

$$t_i = \frac{\sigma_s}{\sqrt{\sigma_n^2 + p}} g_i^* h_i^* q_i \quad i = 1, \ldots n$$

The received signal is

$$\begin{aligned} r = \sum_{i=1}^{n} h_i t_i + w &= \sum_{i=1}^{n} |h_i|^2 \frac{\sigma_s}{\sqrt{\sigma_n^2 + p}} g_i^* q_i + w \\ &= \sum_{i=1}^{n} \frac{\sigma_s}{\sqrt{\sigma_n^2 + p}} \left(|h_i g_i|^2 s + |h_i|^2 g_i^* v_i \right) + w \end{aligned}$$

Thus the signal adds up coherently and the noise non-coherently:

$$\mathrm{SNR} = \frac{\frac{n^2\sigma_s^2}{\sigma_n^2+p}}{\sigma_n^2(1+\frac{n\sigma_s^2}{\sigma_n^2+p})} = \frac{n^2\sigma_s^2 p}{\sigma_n^2(\sigma_n^2+p+n\sigma_s^2)}$$

Assume $p = O(n^{-\epsilon})$ and $n\sigma_s^2 = O(n^{-\epsilon})$ for some $\epsilon > 0$. Then

$$\mathrm{SNR} = \frac{n(n\sigma_s^2)p}{\sigma_n^4} = O(n^{1-2\epsilon}) \quad \text{and} \quad \eta = O(\frac{n^{1-2\epsilon}}{n^{-\epsilon}}) = O(n^{1-\epsilon})$$

Therefore $\epsilon > 1/2$ and so the best we can do is:

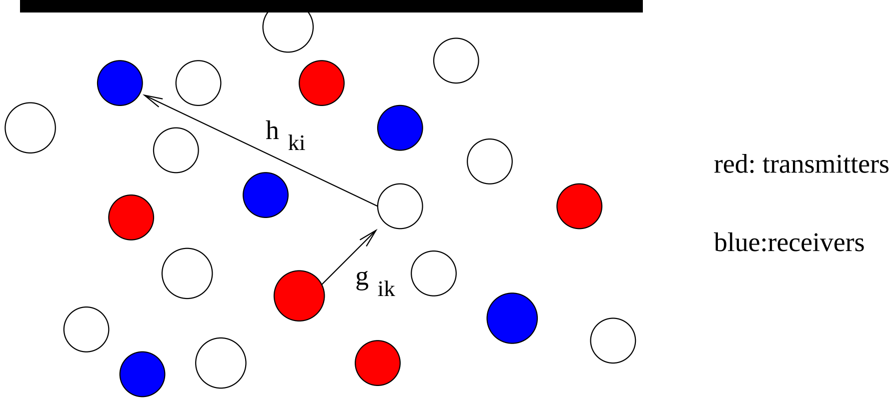
$$\eta = O(\sqrt{n})$$

Power Efficiency of Sensory Networks

- the power efficiency is thus $O(\sqrt{n})$
- compared to a single link, for a fixed transmission rate, the total power consumption in the network can be reduced by a factor of $\frac{1}{\sqrt{n}}$
- this is even better than the $n \times 1$ multi-antenna channel with no channel state information ($O(1)$), though not as good as the $n \times 1$ case with channel state information ($O(n)$)
- the protocol is single, rather than multi-hop, and exploits, rather than avoids, interference
- there is built-in fairness: the transmitter transmits with power $p = O(\frac{1}{\sqrt{n}})$ and all other nodes with power $\sigma_s^2 = O(\frac{1}{n\sqrt{n}})$

Thus, from a power efficiency point of view, it pays off to network!

Ad-Hoc Networks



- n nodes, r users transmitting, r users receiving
- gains from transmitters to each node: $g_{ik}, \quad i = 1, \ldots, r \quad k = 1, \ldots, n$
- gains from each node to receivers: $h_{ki}, \quad k = 1, \ldots, n \quad i = 1, \ldots, r$
- **Assumptions:**
  - $\{g_{ik}, h_{ki}\}$ independent, zero-mean, unit-variance, finite fourth-order moment
  - system synchronous
  - each node k knows its local connections, g_{ik} and h_{kj}, but not the rest of the network

Back to "Listen/Transmit" Protocol

But what if we allow for interference? Let's look at our previous protocol.

1. The listen interval: Each of the r transmit users transmits the signal s_i with power p. All other nodes are silent and measure the signals

$$q_k = \sum_{i=1}^{r} s_i g_{ik} + v_k, \quad i = 1, \ldots, r \quad k = 1, \ldots, n$$

2. The transmit interval: Each of the nodes now has to transmit a signal. Let us assume that all it can transmit is a scaled version of what it has previously received:

$$t_k = d_k q_k, \quad k = 1, \ldots, n$$

such that the transmit power is σ_s^2. But how to choose d_k?

Defining the transmit vector $s = \begin{bmatrix} s_1 & s_2 & \dots & s_r \end{bmatrix}$, the receive vector $y = \begin{bmatrix} y_1 & y_2 & \dots & y_r \end{bmatrix}$ and the matrices

$$G = \begin{bmatrix} g_{11} & g_{12} & \dots & g_{1n} \\ g_{21} & g_{22} & \dots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{r1} & g_{r2} & \dots & g_{rn} \end{bmatrix} \quad , \quad H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1r} \\ h_{21} & h_{22} & \dots & h_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \dots & h_{nr} \end{bmatrix}$$

we may write

$$y = s \cdot G \cdot \mathrm{diag}(d_1, \dots, d_n) \cdot H$$

Note that if $n \geq r^2$, the d_i can be chosen such that

$$G \cdot \mathrm{diag}(d_1, \dots, d_n) \cdot H = I_r, \tag{1}$$

which means that the channel is diagonalized and the interference suppressed!

But this can only be done if each node knows all the network gains so that it can solve (1). This is not allowed.

What to do?

- Let each node estimate each of the r transmitted signals:

$$\hat{s}_i = g_{ik}^* q_k, \quad i = 1, \ldots, r \quad k = 1, \ldots, n$$

  Of course, this will be a lousy estimate since the SINR (signal-to-interference-ratio) is $\frac{p}{\sigma_n^2 + (r-1)p}$.

- Let each node attempt to "coherently add" these estimates:

$$t_k = \frac{1}{\sqrt{r}} \cdot \frac{\sigma_s}{\sqrt{\sigma_n^2 + rp}} \sum_{i=1}^{r} h_{ki}^* \hat{s}_i = \frac{\sigma_s}{\sqrt{r(\sigma_n^2 + rp)}} \sum_{i=1}^{r} h_{ki}^* g_{ik}^* q_k$$

  Note therefore that

$$d_k = \frac{\sigma_s}{\sqrt{r(\sigma_n^2 + rp)}} \sum_{i=1}^{r} h_{ki}^* g_{ik}^*$$

  depends only on *local* knowledge of the network gains.

The received signal at the j-th receiver is:

$$y_j = \sum_{k=1}^{n} t_k h_{kj} + w_j.$$

Close inspection of the received signal reveals that it consists of $n+1$ noise terms (one from each of the n nodes) and nr^2 signal terms, of which

- there are n terms of the desired signal s_j that add up *coherently* (one from each of the n nodes)
- there are $n(r-1)$ terms of the desired signal s_j that add up *non-coherently* ($r-1$ from each of the n nodes)
- there are $nr(r-1)$ interference terms (r from each interferer and each node)

Therefore

$$\mathrm{SINR} = \frac{\frac{(n^2+n(r-1))\sigma_s^2 p}{r(\sigma_n^2+rp)}}{\frac{nr(r-1)\sigma_s^2 p}{r(\sigma_n^2+rp)} + \frac{n\sigma_n^2\sigma_s^2}{r(\sigma_n^2+rp)} + \sigma_n^2} \approx \frac{rpn^2\sigma_r^2}{r^2(\sigma_n^2+rp)(\sigma_n^2+n\sigma_r^2)}$$

Power Efficiency of Ad-Hoc Networks

- a similar analysis to the sensory case shows that, *provided* $r \leq \sqrt{n}$, the power efficiency is $O(\sqrt{n})$
- thus, if we fix the rate for each of the r transmit/receive pairs, then the total power consumption of the network reduces by a factor of $\frac{1}{\sqrt{n}}$.
- this is as good as a sensory network
- rather than diminishing rewards, we are reaping benefits from the increase in the size of the network
- there is built-in fairness: the r transmitters transmit with power $p = O(\frac{1}{\sqrt{n}})$ and all other nodes with power $\sigma_s^2 = O(\frac{r}{n} \cdot \frac{1}{\sqrt{n}})$
- again, the protocol is single, rather than multi-hop, and exploits, rather than avoids, interference

Final Remarks and Further Questions

- if one focuses on power efficiency it apparently does pay off to consider sensory and wireless ad-hoc networks
- the net benefit is larger than the sum of the individual benefits
- to get encouraging asymptotics, rather than attempting to see how the rate scales, the key is to fix the rate and study how the power can be lowered
- interference is, in fact, useful
- **some questions:**
  - are our bounds tight? (we think so, and are very close to a proof)
  - in the ad-hoc case, can the result be extended beyond $r \geq \sqrt{n}$? (this should be difficult, since it requires an improvement of the result of Gupta and Kumar)