Throughput and Delay Analysis of MIMO Broadcast Channels with Partial CSI

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Most of the research on multi-antenna systems has focused on

- **Channel models and information theory**
  - known channels, unknown channels, spatial correlation, wide-band, etc.
- **Space-time codes**
  - BLAST, orthogonal designs, USTM, differential schemes, etc.
- **Algorithms**
  - nulling and canceling, sphere decoding, iterative detection/decoding, etc.

All of this is from a point-to-point perspective. Only recently has there been a focus on the role of multiple antennas in a wireless network.

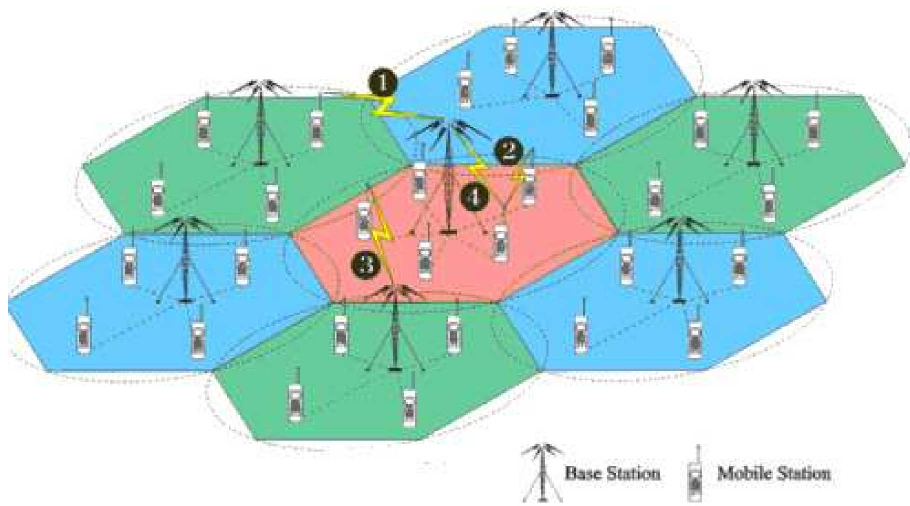
In this talk we will study the role of multiple antennas in broadcast channels, i.e., the *downlink problem*.

Outline

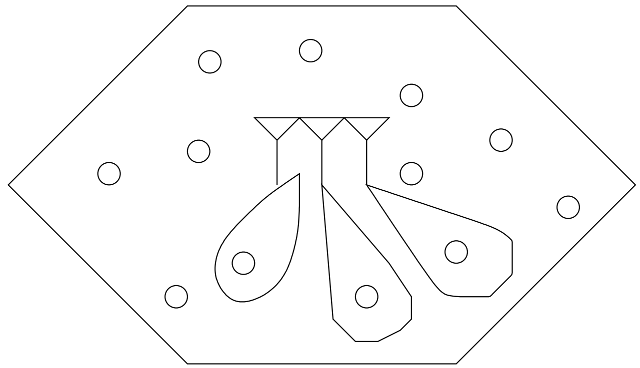
- **Introduction**
- **MIMO broadcast channels**
  - asymptotic sum-rate capacity with and without CSI
  - propose a scheme that requires very little CSI
  - asymptotic analysis
  - fairness issues
  - delay analysis
- **Conclusion**

Introduction to Broadcast Channels

- motivated by down-link scheduling in cellular systems
- to increase network capacity cell sizes are shrinking
- multiple transmit/receive antennas have proven useful in single link communications — what is their role in multi-user networks?



Increasing Cell Capacity with Multiple Antennas



- idea is to exploit *spatial diversity* to transmit simultaneous beams to more than one user (SDMA)
- often requires knowledge of spatial location of different users at base station (also line-of-sight environment)

Assume a system with n users, where the base station has M antennas and each user has N receive antennas.

- The first question we are interested in is how capacity scales with M, N and n.

Multiple Antennas in Single User Links

Consider a single user link with M transmit and N receive antennas. Assume that the environment is rich-scattering.

- Transmitter and receiver know the channel:

$$C = \min(M, N) \log \mathrm{SNR} + O(1)$$

- Only receiver knows the channel (Foschini '97, Telatar '97):

$$C = \min(M, N) \log \mathrm{SNR} + O(1)$$

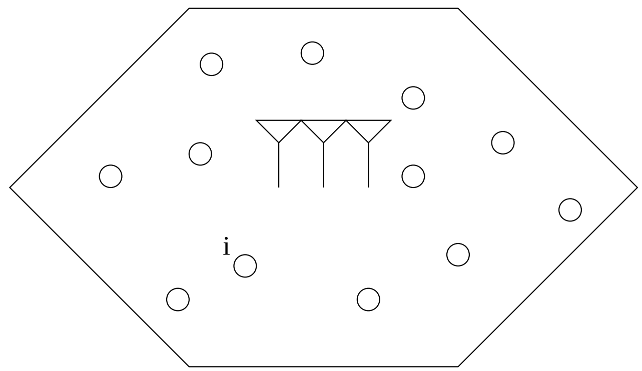
- Neither receive nor transmitter knows the channel (Zheng and Tse '01, Hassibi and Marzetta '01):

$$C = \min(M, N) \left(1 - \frac{\min(M, N)}{T}\right) \log \mathrm{SNR} + O(1)$$

where T is the coherence interval of the channel

Multiple Antennas in Broadcast Channels

Suppose now we have M transmit antennas at the base station.



Assuming $N = 1$, the channel to the i-th user is given by an M-dimensional vector:

$$h_i = \left[\begin{array}{cccc} h_{i1} & h_{i2} & \ldots & h_{iM} \end{array} \right]$$

The h_i are independent random vectors, whose distribution depends on the fading environment.

For Rayleigh fading the entries of h_i are iid complex Gaussian.

Question: *What is the capacity of such a multi-antenna broadcast channel?*

Answer depends on what the transmitter knows about the channels.

Broadcast Channels with full CSI in Tx/Rx

Capacity with full CSI in the transmitter (Caire and Shamai '03, Viswanath and Tse '03, Goldsmith et al. '03):

$$C^{DP} = E\left\{ \max_{\{P_1,\ldots,P_n,\sum P_i = P\}} \log\det\left(I + \sum_{i=1}^{n} h_i^* P_i h_i \right) \right\}.$$

- Achieved by what is known as *dirty paper coding* (Costa '80).
- We have analyzed this for large number of users, fixed M and N, and shown that

$$\lim_{n\to\infty} \frac{C^{DP}}{M \log\log nN} = 1$$

Dirty paper coding presents two problems:

- It requires full channel knowledge at the transmitter and can be sensitive to channel errors
- It is very computationally intensive (VQ at both Tx/Rx)

Capacity of MIMO BC with no CSI in Tx

What if we assume no CSI in Tx?

- It is straightforward to show that the Gaussian MIMO BC with no CSI in the transmitter is *degraded* no matter whether the receivers have CSI or not (Amraoui et al '03, Sharif and Hassibi '03).
- Therefore, when the users have the same number of antennas, superposition coding is the same as time sharing.
- Furthermore, assuming $N = 1$, the sum rate capacity is

  $$C = \log M + O(1)$$

  independent of n! (Assuming a fixed transmit power per antenna.)

Thus, when $N = 1$, lack of knowledge of the channel coefficients brings us down from $M \log \log n$ to $\log M$.

- full channel-state-information (CSI) unreasonable
- multiple antennas do not buy us anything if no CSI available

What to do?

- is there any critical side information in the transmitter that can fill this gap?

Send Random Beams

Choose M random orthonormal vectors ϕ_m, $m = 1, \ldots, M$ (according to an isotropic distribution). At time t, the m-th vector is multiplied by the signal $s_m(t)$ ao that the transmitted signal is

$$S(t) = \sum_{m=1}^{M} \phi_m s_m(t), \qquad t = 1, \ldots, T$$

where T is less than the coherence interval of the channel.

After T channel uses we independently choose another isotropic set of orthonormal vectors $\{\phi_m\}$, and so on.

In other words, we are transmitting M random beams.

Each receiver $i = 1, \ldots, n$ therefore can compute the following M SINRs

$$\text{SINR}_{im} = \frac{|h_i \phi_m|^2}{1/\text{SNR} + \sum_{n \neq m} |h_i \phi_n|^2}$$

Of course, on average the SINR is (roughly) $\frac{1}{1/\text{SNR}+M-1}$, and so if we randomly assign beams to users we get

$$C \approx M \log \left(1 + \frac{1}{1/\text{SNR} + M - 1} \right) < \frac{M}{1/\text{SNR} + M - 1},$$

which is pretty lousy.

So what is the point?

Exploit Multi-User Diversity

Suppose now each user (or, in fact, only those who get favorable SINRs) **feeds back to the transmitter its best SINR**.

Rather than randomly assign the beams, the transmitter assigns signal s_m to the user with the best SINR for that signal. Therefore

$$C = E \sum_{m=1}^{M} \log \left(1 + \max_{i=1,\dots,n} \frac{|h_i \phi_m|^2}{1/\text{SNR} + \sum_{n \neq m} |h_i \phi_n|^2} \right)$$

Due to the symmetry of all the random variables involved:

$$C = ME \log \left(1 + \max_{i=1,\dots,n} \frac{|h_i \phi_1|^2}{1/\text{SNR} + \sum_{n \neq 1} |h_i \phi_n|^2} \right)$$

Note that the random variables

$$\frac{|h_i \phi_1|^2}{1/\text{SNR} + \sum_{n \neq 1} |h_i \phi_n|^2}$$

are ratios of independent Gaussians, and themselves are independent.

Maximum of n IID Random Variables

Theorem 1 (Frechet, 1927, v. Mises 1947, Uzgoren 1954) *Let x_i, $i = 1, \ldots, n$ be iid random variables with distribution $p(x)$. Then if,*

$$\lim_{x \to \infty} -\frac{p(x)}{p'(x)} = c > 0,$$

then

$$Prob\left(\frac{|x_{\max} - \frac{1}{c}\log n|}{\log\log n} > \epsilon\right) < \frac{\delta}{(\log n)^{\alpha}}.$$

In our case, the condition of the theorem is met with $c = 1/\mathrm{SNR}$.

Therefore, for M fixed and $n \to \infty$

$$C = M \log\log n + O(\log\log\log n)$$

Thus, multi-user diversity buys us a lot!

Some Remarks

- Using partial CSI, our scheme achieves $M \log \log n$, the same asymptotic capacity as having full CSI.
- If we have log-normal shadowing

$$C = M\sqrt{\log n} + O(\sqrt{\log \log n}).$$

- Compared to the full CSI case where each user had to feedback $2M$ complex numbers, here each user need only feed back its *best* SINR and the corresponding index.
  - In fact, not all users need to do this. If only users whose best SINR exceeds a threshold, η, say, feed it back then

$$C \geq \left(1 - F^{Mn}(\eta)\right) M \log \log n + O(\log \log \log n),$$

    where $F(\cdot)$ is the CDF of the SINR. This reduces the feedback by a factor of $\frac{1}{1-F(\eta)}$.

How Large can M Be?

Our result required that M be fixed and $n \to \infty$.

In this case, $\mathrm{SINR}_{\max}$ was obtained when $\sum_{j \neq m} |h_j \phi_m|^2 \approx 0$ and $|h_i \phi_m|^2 \approx \log n$.

However, in practice, we will have a large (but finite n) and so it is useful to know how large M can be to retain a linear increase in capacity.

Result: Let $M = \alpha \log n$. Then as $n \to \infty$, we have $\mathrm{SINR}_{\max} \to 1/\alpha$ and

$$C = M \log\left(1 + \frac{1}{\alpha}\right) + O(1)$$

Moreover, if $\frac{M}{\log n} \to \infty$, then $\frac{C}{M} \to 0$.

Sum Rate Throughput of Random Beamforming

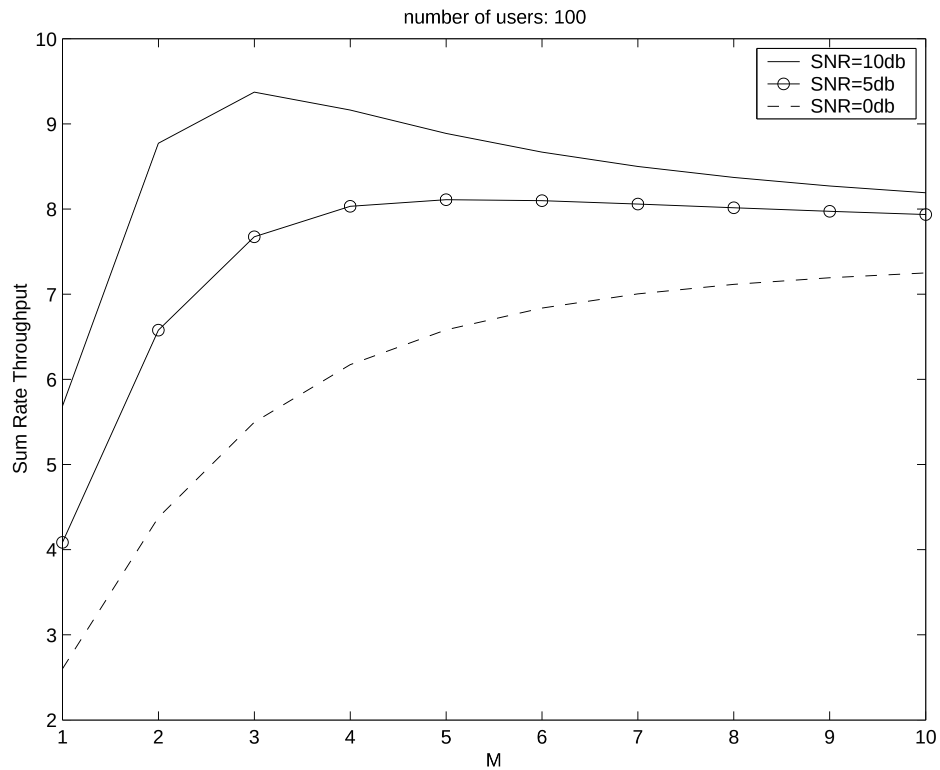


Figure 1: Sum rate throughput for a system with $n = 100$ users and for different number of transmit antennas

What if We Add Multiple Receive Antennas?

Suppose we have $N > 1$ receive antennas. There are three possibilities:

- assign one beam to user (feed back capacity)

$$C = M \log\log n + O(\log\log\log n)$$

- assign M/N beams to user (feed back capacity)

$$C = M \log\log n/N + O(\log\log\log n)$$

- assign one beam to user (treat each antenna independently)

$$C = M \log\log nN + O(\log\log\log n)$$

The last is the best. In any case, multiple receive antennas do not substantially increase the network capacity.

- However, if some fraction of the users have N receive antennas they will receive N-times the rate.

Fairness of the System

- So far we have assumed that the network is homogenous
- Most networks are heterogenous, in the sense that the SNRs for the different users are different
- Thus, if we transmit to the most favorable users, the system may be dominated by the users with the highest SNR
- This is certainly true for single-transmit-antenna systems

Single Antenna Multi-User Fairness

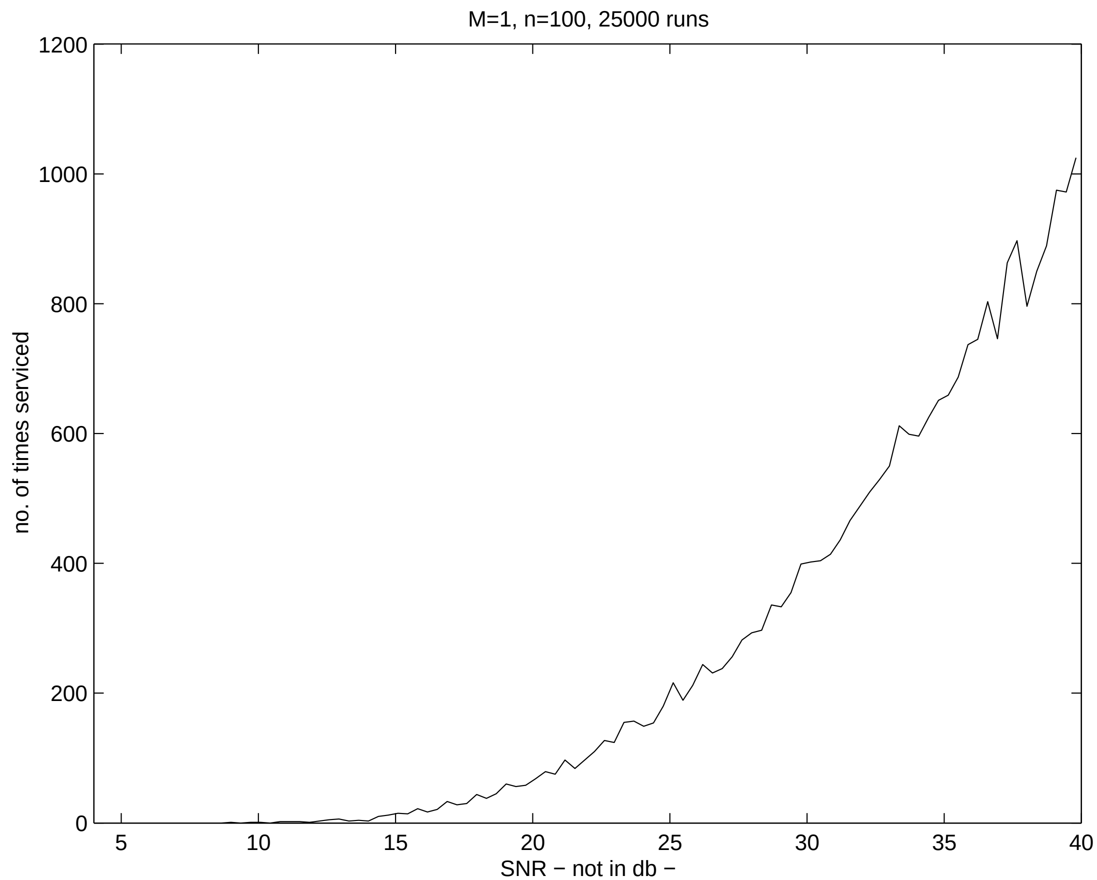


Figure 2: $M = 1$, $n = 100$, SNR 4-40 (6-16db), 25000 runs

Fairness - Continued

In a heterogenous network, the SINRs now become

$$\text{SINR}_{im} = \frac{|h_i \phi_m|^2}{1/\text{SNR}_i + \sum_{n \neq m} |h_i \phi_n|^2}$$

Note that, if the SNR_is are high enough, or if the number of transmit antennas is large enough, then the system is interference-dominated which implies *built-in fairness*.

Result:

$$P(\text{choosing user with SNR}_{min}) \geq \frac{1}{n} e^{-\left(\frac{1}{SNR_{min}} - \frac{1}{SNR_{max}}\right)\left(e^{\frac{\log n}{M}} - 1\right)},$$

This further illustrates the benefits of having $M = \alpha \log n$.

Multi-Antenna Multi-User Fairness

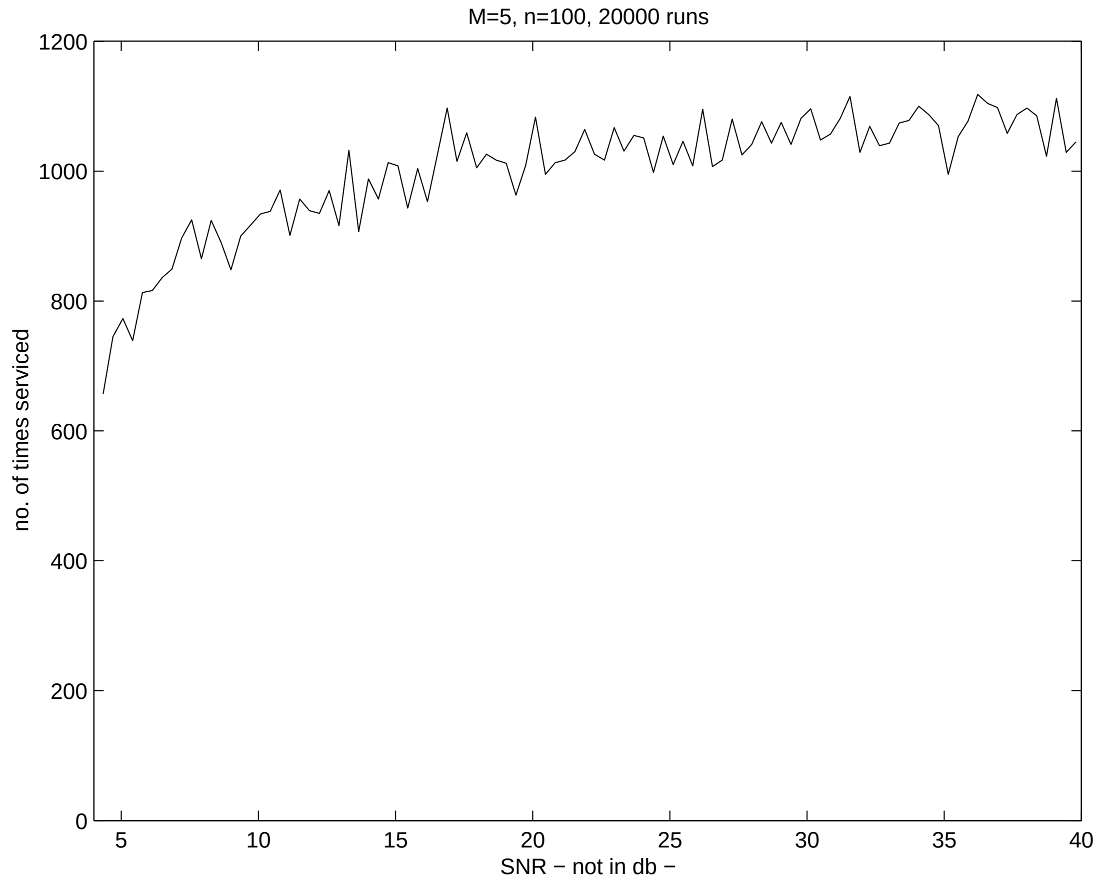


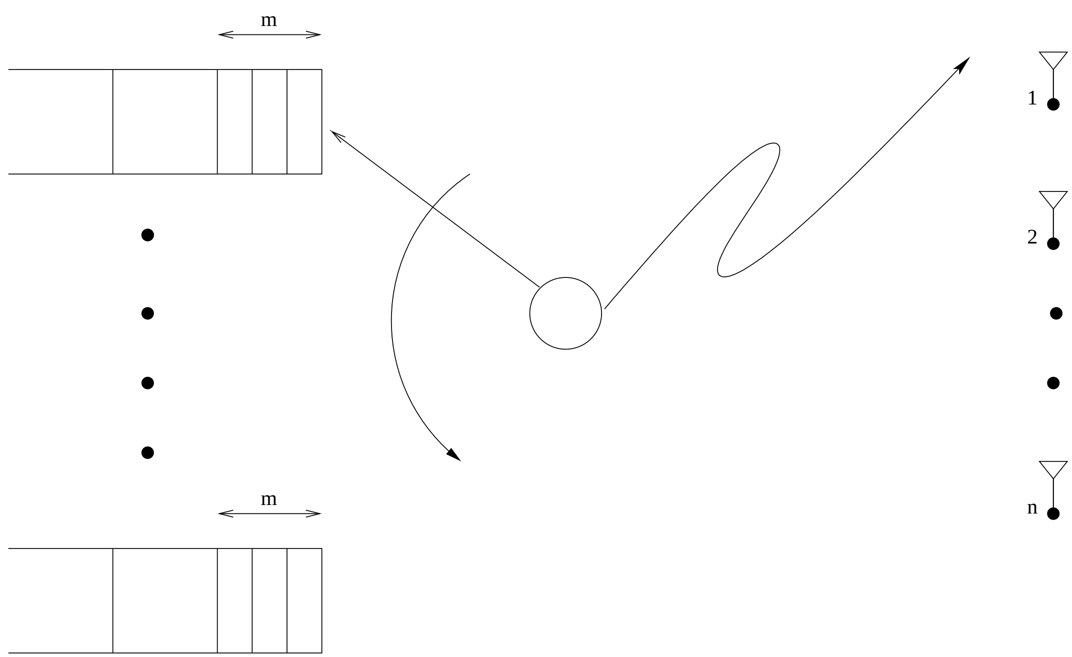
Figure 3: $M = 5$, $n = 100$, SNR 4-40 (6-16db), 20,000 runs

Delay Guarantee in Broadcast Channels

- So far we have considered capacity and fairness.
- The fairness we considered is really "long-term" fairness.
- We now want to focus on "short-term" fairness, which really means the study of delay.

Delay in SISO Broadcast Channels

- For simplicity, we start with SISO BC systems.
- We consider a packet-based transmission scheme.
- Delay, $D_{m,n}$, is defined as the minimum number of channel uses that guarantees that all n users successfully receive m packets.



Assumptions

- Channel is block fading, and is independent from packet to packet.
- Assume all the packets carry C_0 information bits.
- A packet is dropped if outage occurs, i.e., if the instantaneous capacity C goes below C_0 at the time of the transmission.
- The throughput therefore is $R = C_0 \Pr\{C \geq C_0\}$.
- The throughput-optimal strategy is the one that transmits to the user with the best channel condition.

Here are the main questions:

1. How bad is the delay for throughput-optimal strategies?
2. Can we reduce the delay at the expense of a small throughput hit?
3. What is the effect of temporal channel correlation?
4. What happens if we have multiple transmit antennas?

Delay: Round Robin vs Throughput-Optimal

- The minimum delay is mn, and is achieved by a round-robin scheme.
- Given the error probability $P_e = \Pr(C \leq C_0)$, the round-robin scheme achieves a rate of $R = C_0(1 - P_e)$.
- Similarly for the throughput-optimal, the rate would be

$$R = C_0' \Pr(\max_{i=1,\ldots,n} C_i \geq C_0').$$

- We are interested in characterizing the delay of this scheme.

Asymptotic Analysis of the Delay

1. For n fixed and $m \to \infty$,

$$E(D_{m,n}) = mn + O(n \log m),$$

which clearly states that the system is long-term fair.

2. For m fixed and $n \to \infty$, we have

$$E(D_{m,n}) = n \log n + O(n \log\log n) \quad , \quad \sigma^2(D_{m,n}) = O(n^2)$$

3. For $m = \log n$ and $n \to \infty$, we have

$$E(D_{m,n}) = 3.126 \underbrace{n \log n}_{mn} + O(n \log\log n).$$

4. For $m = (\log n)^r$ where $r > 1$ is fixed and $n \to \infty$, then

$$E(D_{m,n}) = \underbrace{n(\log n)^r}_{mn} + O(n \log n),$$

which demonstrates how long it takes for the system to become fair.

Role of Multiple Antennas

Assuming that the channels fade independently from packet to packet, with M transmit antennas for fixed m and $n \to \infty$, we have

$$E(D_{m,n}) = \frac{n \log n}{M + \log M} + O(n \log \log n),$$

which is not much of an improvement over a system that is M times faster.

However, multiple antennas improve the delay in two important ways:

- When the channels have high correlation from packet to packet, the use of independent random beams can decorrelate the SINRs
- When $M = \log n$, using random beams makes all users equally likely to be chosen.

Summary

- scaling laws for sum rate of MIMO BC
  - full CSI in TX, capacity scales like $M \log\log n$
  - no CSI in TX, capacity scales like $\log M$
- proposed a simple scheme requiring very little feedback
  - throughput scales like $M \log\log n$
  - linear growth for $M = O(\log n)$
  - exploits multi-user diversity, but also has built-in fairness
  - optimal number of transmit antennas appears to be $\alpha \log n$
- analyzed the delay in broadcast channels
  - delay for the optimal-throughput scheduling
  - role of multiple antennas