Some Asymptotic Results for Wireless Networks

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Outline

- **Introduction**
- **Cellular Networks** (joint work with M. Sharif)
  - capacity of multi-antenna broadcast channels with partial CSI
  - fairness issues
- **Sensory and Ad-Hoc Wireles Networks** (joint work with A. Dana)
  - power efficiency
- **Conclusion**

Wireless Networks

- wireless is a *shared* medium
  - network is a fully-connected graph
  - bandwidth is precious
- distinguishing features
  - interference
  - path-loss
  - fading

Traditional methods attempt to combat these “deficiencies”.

A better approach is to *exploit* them.

Brief History

- First wireless networks consisted of a single base station with large coverage in an urban center, along with mobiles (due to interference, very few could be accomodated)
- cellular networks
  - theory developed in Bell Labs in 60's and 70's
  - first system deployed in early 80's (AMPS)
  - key: exploit path-loss to allow for frequency reuse
  - to increase capacity, cells are shrinking (micro-cells, etc.)
  - FDMA, TDMA, CDMA
- WLAN's (802.11's)
- mobile Internet, etc.

Sensory Networks and Ad-Hoc Networks

Sensory Networks:

- applications: environmental, surveillance, military, etc.
- sensor nodes are simple, cheap and consume very little power
- all information intended for a single receiver
- sensors may only need to occasionally transmit information

Ad-Hoc Networks:

- no infrastructure
- users "somehow" cooperate in a distributed fashion to communicate
- can have many users acting simultaneously as transmitters or receivers

Basic Problem to be Addressed

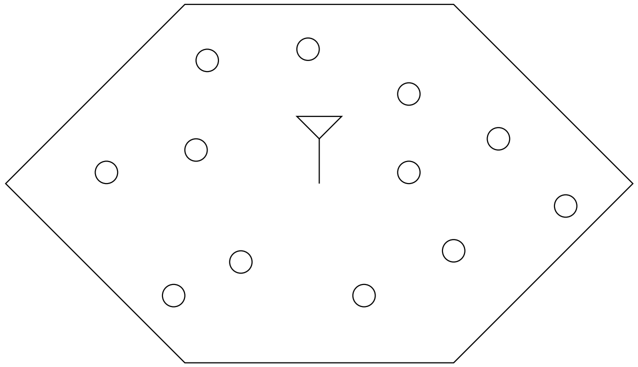
For cellular, sensory and ad-hoc wireless networks we are interested in how various performance measures, such as capacity and power efficiency, scale in the number of users/sensors.

This is related to **Network Information Theory**:

- single-user information theory well-understood (Shannon)
- multi-user information theory, however, is not
  - computing the capacity of even a three-node network is open
  - situation reminiscent of mechanics where three-body problems have not been solved

This is why we will focus on asymptotic results for *large* networks (analogous to statistical mechanics where we have many bodies)

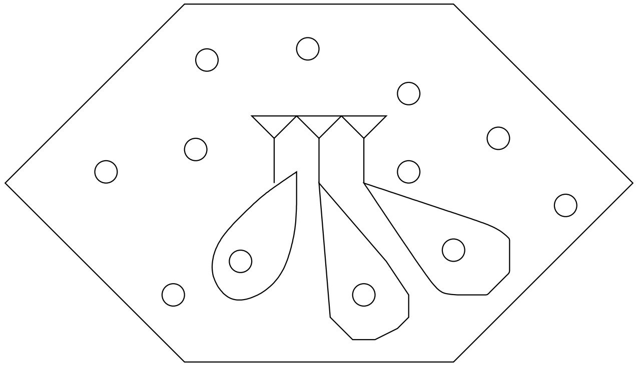
Cellular Networks



- capacity per cell is fixed (depends on bandwidth and transmit power)
- TDMA, FDMA, CDMA all essentially give same capacity
- to increase network capacity cell sizes are shrinking

Under ideal conditions, there exist sophisticated techniques where capacity scales by $\log n$ (n is number of users)

Increasing Cell Capacity with Multiple Antennas



- idea is to exploit *spatial diversity* to transmit simultaneous beams to more than one user
- requires knowledge of spatial location of different users at base station (also line-of-sight environment)
- some companies attempted this in the 90's (e.g., ArrayComm) but it never really caught on

Multiple Antennas in Single User Links

Consider a single user link with M transmit and N receive antennas. Assume that the environment is rich-scattering.

- Transmitter and receiver know the channel:

$$C = \min(M, N) \log \mathrm{SNR} + O(1)$$

- Only receiver knows the channel (Foschini '97, Telatar '97):

$$C = \min(M, N) \log \mathrm{SNR} + O(1)$$

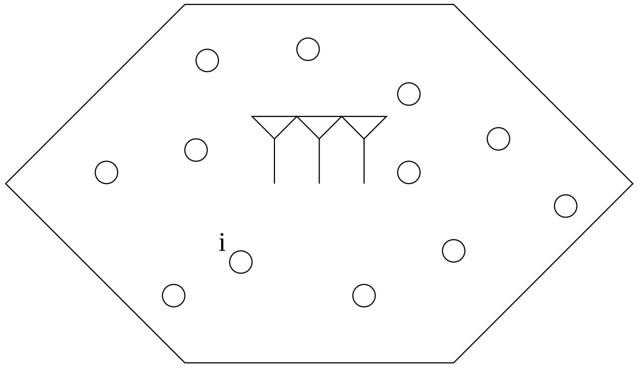
- Neither receive nor transmitter knows the channel (Zheng and Tse '01, Hassibi and Marzetta '01):

$$C = \min(M, N) \left(1 - \frac{\min(M, N)}{T}\right) \log \mathrm{SNR} + O(1)$$

  where T is the coherence interval of the channel

Multiple Antennas in Broadcast Channels

Suppose now we have M transmit antennas at the base station.



The channel to the i-th user is given by an M-dimensional vector:

$$h_i = \left[\begin{array}{cccc} h_{i1} & h_{i2} & \ldots & h_{iM} \end{array} \right]$$

The h_i are independent random vectors, whose distribution depends on the fading environment.

For Rayleigh fading the entries of h_i are iid complex Gaussian.

So what is the capacity of such a multi-antenna broadcast channel?

- Channel coefficients known to transmitter (Shamai and Caire '02):

$$C = \max_{\Pi, \mathrm{tr} \sum_i R_i = P} \sum_{i=1}^{n} E \log \det \left(I_M + \left(\sigma^2 I_M + h_i^* \sum_{j=1}^{i-1} R_j h_i \right)^{-1} h_i^* R_i h_i \right)$$

  - Achieved by what is known as *dirty paper coding* (Costa '80).
  - Asymptotically, for fixed M and $n \to \infty$, this behaves as

$$C = M \log n + O(1)$$

  - The problem is that this requires full channel knowledge at the transmitter and is very sensitive to channel errors
  - Dirty paper coding is very computationally-intensive (both the transmitter and receiver need to do vector quantization), although there exist simplified schemes such as Tomlinson-Harashima precoding with reasonable performance

- Channel coefficients unknown to transmitter:

$$C = \log M + O(1)$$

independent of n! (Assuming fixed power per transmit antenna.)

Thus, lack of knowledge of the channel coefficients brings us down from $M \log n$ to $\log M$.

- full channel-state-information (CSI) unreasonable
- multiple antennas do not buy us much if no CSI available

What to do?

Send Random Beams

Choose M random orthonormal vectors ϕ_m, $m = 1, \dots, M$ (according to an isotropic distribution). At time t, the m-th vector is multiplied by the signal $s_m(t)$ ao that the transmitted signal is

$$S(t) = \sum_{m=1}^{M} \phi_m s_m(t), \qquad t = 1, \dots, T$$

where T is less than the coherence interval of the channel.

After T channel uses we independently choose another isotropic set of orthonormal vectors $\{\phi_m\}$, and so on.

In other words, we are transmitting M random beams.

Each receiver $i = 1, \ldots, n$ therefore can compute the following M SINRs

$$\mathrm{SINR}_{im} = \frac{|h_i \phi_m|^2}{1/\mathrm{SNR} + \sum_{n \neq m} |h_i \phi_n|^2}$$

Of course, on average the SINR is (roughly) $\frac{1}{1/\mathrm{SNR}+M-1}$, and so if we randomly assign beams to users we get

$$C \approx M \log \left(1 + \frac{1}{1/\mathrm{SNR} + M - 1}\right) < \frac{M}{1/\mathrm{SNR} + M - 1},$$

which is pretty lousy.

So what is the point?

Exploit Multi-User Diversity

Suppose now each user (or, in fact, only those who get favorable SINRs) feeds back to the transmitter its best SINR.

Rather than randomly assign the beams, the transmitter assigns signal s_m to the user with the best SINR for that signal. Therefore

$$C = E \sum_{m=1}^{M} \log\left(1 + \max_{i=1,\ldots,n} \frac{|h_i \phi_m|^2}{1/\mathrm{SNR} + \sum_{n \neq m} |h_i \phi_n|^2}\right)$$

Due to the symmetry of all the random variables involved:

$$C = M E \log\left(1 + \max_{i=1,\ldots,n} \frac{|h_i \phi_1|^2}{1/\mathrm{SNR} + \sum_{n \neq 1} |h_i \phi_n|^2}\right)$$

Note that the random variables

$$\frac{|h_i \phi_1|^2}{1/\mathrm{SNR} + \sum_{n \neq 1} |h_i \phi_n|^2}$$

are ratios of independent Gaussians, and themselves are independent.

Maximum of n IID Random Variables

Theorem 1 (Frechet, 1927, v. Mises 1947, Uzgoren 1954) *Let x_i, $i = 1, \ldots, n$ be iid random variables with distribution $p(x)$. Then if,*

$$\lim_{x \to \infty} -\frac{p(x)}{p'(x)} = c > 0,$$

then

$$Prob\left(\frac{|x_{\max} - \frac{1}{c}\log n|}{\log\log n} > \epsilon\right) < \frac{\delta}{(\log n)^{\alpha}}.$$

In our case, the condition of the theorem is met with $c = 1/\mathrm{SNR}$.

Therefore, for M fixed and $n \to \infty$

$$C = M \log\log n + O(\log\log\log n)$$

Thus, multi-user diversity buys us a lot!

Some Remarks

- Using partial CSI, our scheme achieves $M \log \log n$, as opposed to $M \log n$ for full CSI and $\log M$ for no CSI.
- If we have log-normal shadowing
$$C = M\sqrt{\log n} + O(\sqrt{\log \log n}).$$
- Compared to the full CSI case where each user had to feedback M channel coefficients, here each user need only feed back its *best* SINR and the corresponding index.
  - In fact, not all users need to do this. If only users whose best SINR exceeds a threshold, η, say, feed it back then
$$C \geq \left(1 - F^{Mn}(\eta)\right) M \log \log n + O(\log \log \log n),$$
    where $F(\cdot)$ is the CDF of the SINR. This reduces the feedback by a factor of $\frac{1}{1-F(\eta)}$.

How Large can M Be?

Our result required that M be fixed and $n \to \infty$.

In this case, $\text{SINR}_{\max}$ was obtained when $\sum_{j \neq m} |h_j \phi_m|^2 \approx 0$ and $|h_i \phi_m|^2 \approx \log n$.

However, in practice, we will have a large (but finite n) and so it is useful to know how large M can be to retain a linear increase in capacity.

Result: Let $M = \alpha \log n$. Then as $n \to \infty$, we have $\text{SINR}_{\max} \to 1/\alpha$ and

$$C = M \log \left(1 + \frac{1}{\alpha}\right) + O(1)$$

Moreover, if $\frac{M}{\log n} \to \infty$, then $\frac{C}{M} \to 0$.

What if We Add Multiple Receive Antennas?

Suppose we have $N > 1$ receive antennas. There are three possibilities:

- assign one beam to user (feed back capacity)

$$C = M \log\log n + O(\log\log\log n)$$

- assign M/N beams to user (feed back capacity)

$$C = M \log\log n/N + O(\log\log\log n)$$

- assign one beam to user (treat each antenna independently)

$$C = M \log\log nN + O(\log\log\log n)$$

The last is the best. In any case, multiple receive antennas do not substantially increase the network capacity.

- However, if some fraction of the users have N receive antennas they will receive N-times the rate.

Fairness of the System

- So far we have assumed that the network is homogenous
- Most networks are heterogenous, in the sense that the SNRs for the different users are different
- Thus, if we transmit to the most favorable users, the system may be dominated by the users with the highest SNR
- This is certainly true for single-transmit-antenna systems

Single Antenna Multi-User Fairness

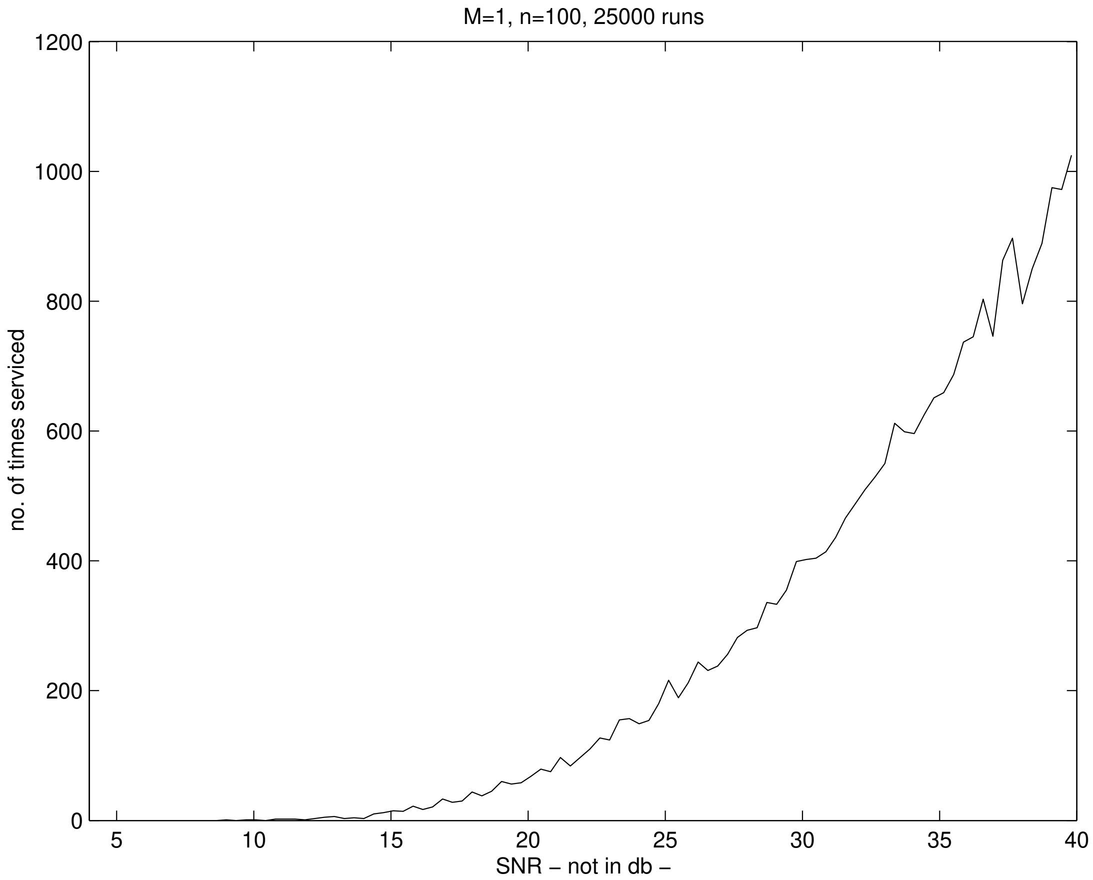


Figure 1: $M = 1$, $n = 100$, SNR 4-40 (6-16db), 25000 runs

Fairness - Continued

In a heterogenous network, the SINRs now become

$$\mathrm{SINR}_{im} = \frac{|h_i \phi_m|^2}{1/\mathrm{SNR}_i + \sum_{n \neq m} |h_i \phi_n|^2}$$

Note that, if the SNR_is are high enough, or if the number of transmit antennas is large enough, then the system is interference-dominated which implies *built-in fairness.*

In fact, it can be shown that

$$P(\text{choosing user with } \mathrm{SNR}_{min}) \geq \frac{1}{n} e^{-\left(\frac{1}{SNR_{min}} - \frac{1}{SNR_{max}}\right)\frac{\log n}{M}},$$

which further illustrates the benefits of having $M = \alpha \log n$.

Multi-Antenna Multi-User Fairness

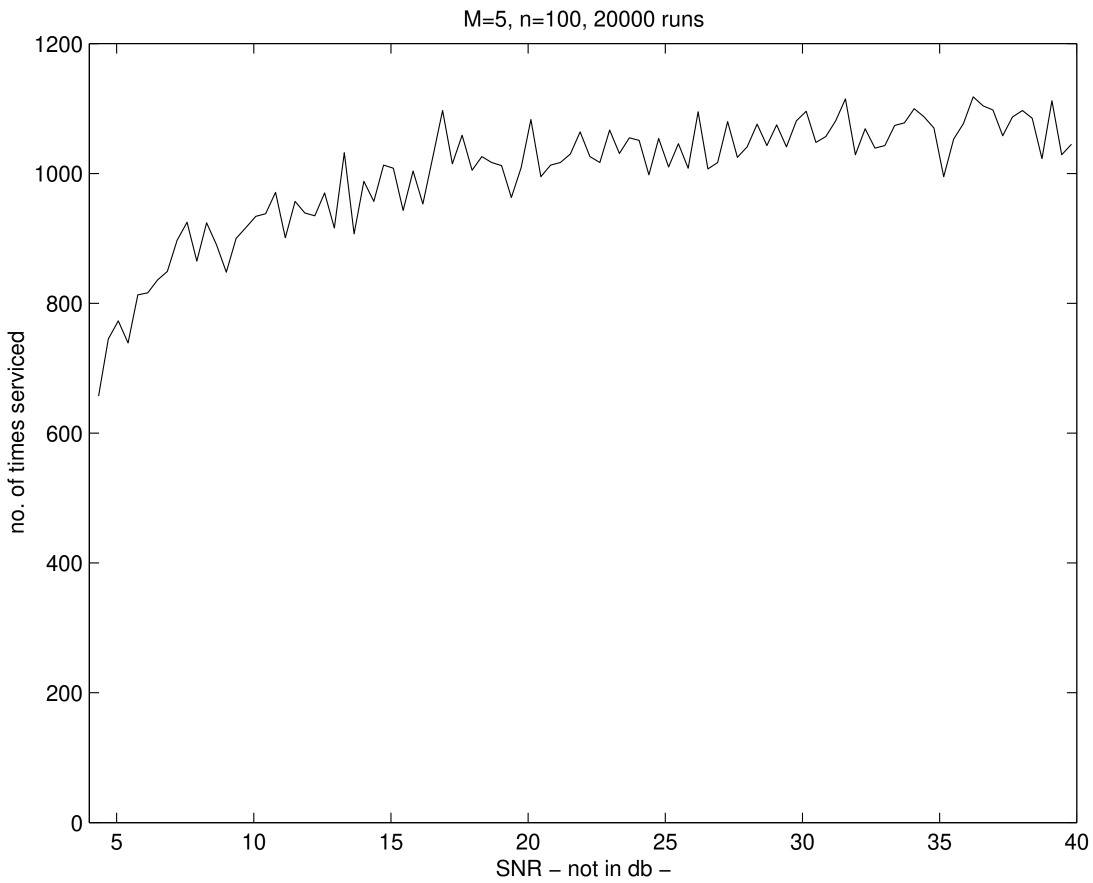


Figure 2: $M = 5$, $n = 100$, SNR 4-40 (6-16db), 20,000 runs

Sensory Networks and Ad-Hoc Networks

Sensory Networks:

- applications: environmental, surveillance, military, etc.
- sensor nodes are simple, cheap and consume very little power
- all information intended for a single receiver
- sensors may only need to occasionally transmit information

Ad-Hoc Networks:

- no infrastructure
- users "somehow" cooperate in a distributed fashion to communicate
- can have many users acting simultaneously as transmitters or receivers

Sensory and Ad-Hoc Networks: Scaling Laws

How does the capacity of such networks scale in n, the number of nodes?

- for sensory networks capacity scales as $O(\log n)$ (Gastpar and Vetterli, 2002)
- for ad-hoc networks capacity scales as $O(\sqrt{n})$ (Gupta and Kumar, 2000)

Both are overall discouraging results:

- in sensory networks, the capacity per participating node is $O(\frac{\log n}{n})$
- in ad-hoc networks, the capacity per participating node is $O(\frac{1}{\sqrt{n}})$

In either case, these are diminishing returns for the nodes' energy investment and participation in the communications

Power Efficiency

- in wireless networks, especially sensory networks, power consumption is a major bottleneck
- this has led many researchers[a] to study bits/energy

Following Verdu (2002), we shall rather look at (bits/sec)/(energy/sec), i.e., at *power efficiency* defined as

$$\eta = \frac{\text{capacity}}{\text{transmit power}}$$

For an AWGN channel this is

$$\eta = \frac{\log(1 + \sigma_s^2/\sigma_n^2)}{\sigma_s^2}$$

[a] McEliece et al, Ephremides et al, El Gamal et al, Goldsmith et al, etc.

Clearly,

- as $\sigma_s^2 \to \infty$:

$$\eta \to \frac{\log \sigma_s^2}{\sigma_s^2} \to 0,$$

  which means we are power in-efficient at high SNR

- as $\sigma_s^2 \to 0$:

$$\eta = \frac{\log(1 + \sigma_s^2/\sigma_n^2)}{\sigma_s^2} \to \frac{\log e}{\sigma_n^2} = O(1),$$

  which means we are power efficient at low SNR

This was all from a single-user perspective.

What happens in a network setting?...

Multi-Antenna Power Efficiency (Verdu et al '02)

To gain some insight, consider an n-transmit single-receive multi-antenna channel, with channel matrix:

$$H = \begin{bmatrix} h_1 & h_2 & \ldots & h_n \end{bmatrix} \quad , \quad E|h_i|^2 = 1$$

- if the channel matrix is *known* to the transmitter (beamforming):

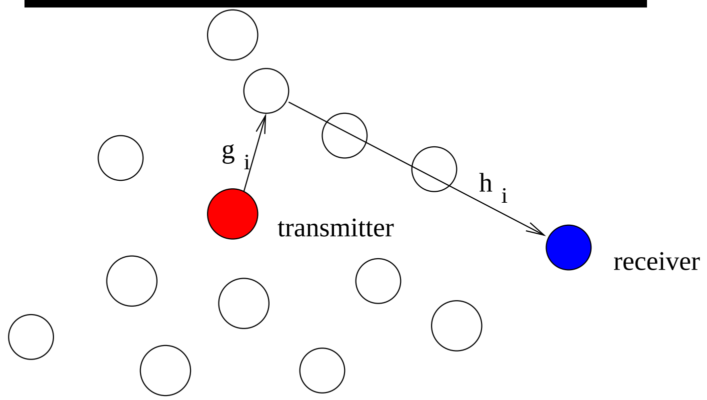
$$\eta = \frac{E \log \left(1 + \frac{\sigma_s^2}{\sigma_n^2} (\sum_{i=1}^{n} |h_i|^2)^2\right)}{n\sigma_s^2} \to \frac{n \log e}{\sigma_n^2} = O(n)$$

- if the channel matrix *unknown* to the transmitter (Foschini-Telatar):

$$\eta = \frac{E \log \left(1 + \frac{\sigma_s^2}{\sigma_n^2} \sum_{i=1}^{n} |h_i|^2\right)}{n\sigma_s^2} \to \frac{\log e}{\sigma_n^2} = O(1)$$

From a power efficiency point of view, it only pays off to have multiple transmit antennas if the channel is known at the transmitter.

Sensory Networks



- gains from transmitter to each node: $g_i, \quad i = 1, \ldots, n$
- gains from each node to receiver: $h_i, \quad i = 1, \ldots, n$
- **Assumptions:**
  - $\{g_i, h_i\}$ are subject to independent fading. Averaged over the fading and the point placement of the nodes, they are unit-variance with finite fourth-order moment
  - system is synchronous
  - each node i knows its local connections, g_i and h_i, but not the rest of the network

A "Listen and Transmit" Protocol

Communication is divided into two intervals:

1. The *listen* interval: the transmitter transmits the signal s, with power p, and all other nodes are silent, but listen:

$$q_i = g_i s + v_i, \quad i = 1, \ldots, n$$

2. The *transmit* interval: each node transmits with power σ_s^2 the signal

$$t_i = \frac{\sigma_s}{\sqrt{\sigma_n^2 + p}} g_i^* h_i^* q_i \quad i = 1, \ldots n$$

The received signal is

$$\begin{aligned} r = \sum_{i=1}^{n} h_i t_i + w \quad &= \quad \sum_{i=1}^{n} |h_i|^2 \frac{\sigma_s}{\sqrt{\sigma_n^2 + p}} g_i^* q_i + w \\ &= \quad \sum_{i=1}^{n} \frac{\sigma_s}{\sqrt{\sigma_n^2 + p}} \left(|h_i g_i|^2 s + |h_i|^2 g_i^* v_i \right) + w \end{aligned}$$

Thus the signal adds up coherently and the noise non-coherently:

$$\text{SNR} = \frac{\frac{n^2\sigma_s^2}{\sigma_n^2+p}}{\sigma_n^2(1+\frac{n\sigma_s^2}{\sigma_n^2+p})} = \frac{n^2\sigma_s^2 p}{\sigma_n^2(\sigma_n^2+p+n\sigma_s^2)}$$

Assume $p = O(n^{-\epsilon})$ and $n\sigma_s^2 = O(n^{-\epsilon})$ for some $\epsilon > 0$. Then

$$\text{SNR} = \frac{n(n\sigma_s^2)p}{\sigma_n^4} = O(n^{1-2\epsilon}) \quad \text{and} \quad \eta = O(\frac{n^{1-2\epsilon}}{n^{-\epsilon}}) = O(n^{1-\epsilon})$$

Therefore $\epsilon > 1/2$ and so the best we can do is:

$$\eta = O(\sqrt{n})$$

Precise Statement

The above argument was very rough. Here is the precise statement.

Theorem 2 *Consider a fixed area A and randomly select n nodes and a transmit/receive pair in this area. Then*

$$Prob\left(\eta > k_1\sqrt{n}\right) > 1 - \frac{k_2}{n},$$

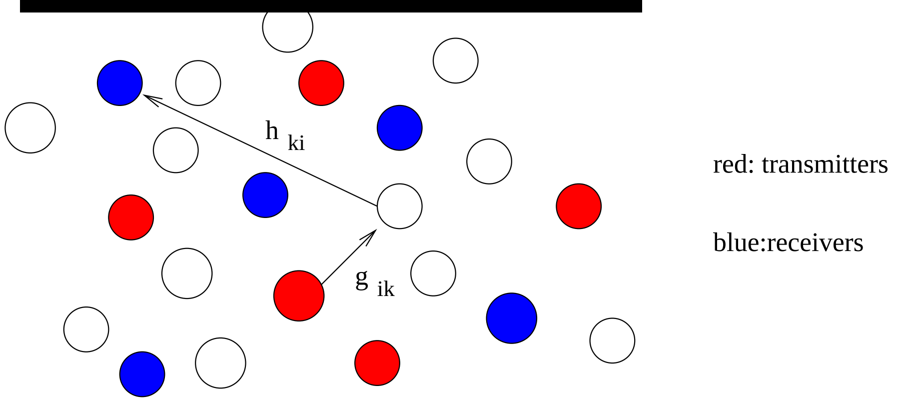
where k_1 and k_2 are positive constants independent of n, but that depend on A and the fading statistics.

Power Efficiency of Sensory Networks

- the power efficiency is thus $O(\sqrt{n})$
- compared to a single link, for a fixed transmission rate, the total power consumption in the network can be reduced by a factor of $\frac{1}{\sqrt{n}}$
- this is even better than the $n \times 1$ multi-antenna channel with no channel state information ($O(1)$), though not as good as the $n \times 1$ case with channel state information ($O(n)$)
- the protocol is double-hop and exploits, rather than avoids, interference (it essentially does distributed beam-forming)
- there is built-in fairness: the transmitter transmits with power $p = O(\frac{1}{\sqrt{n}})$ and all other nodes with power $\sigma_s^2 = O(\frac{1}{n\sqrt{n}})$

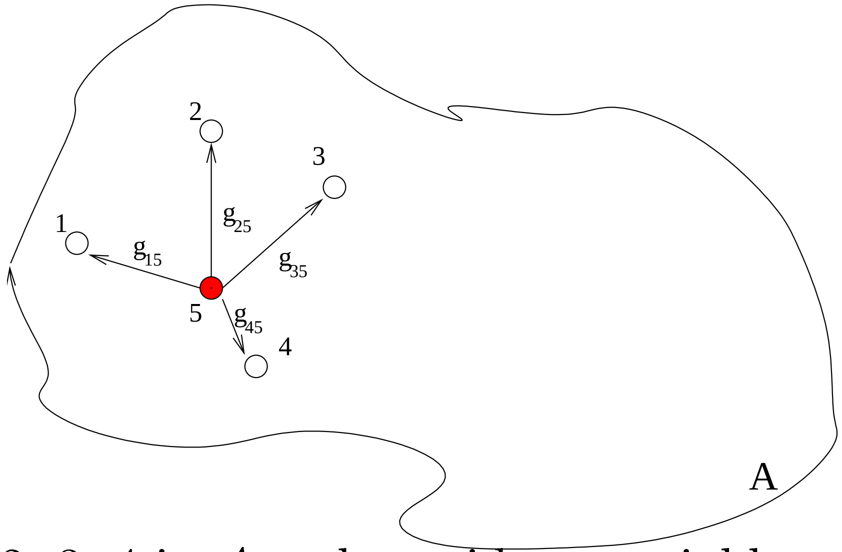
Thus, from a power efficiency point of view, it pays off to network!

Ad-Hoc Networks



- n nodes, $r \leq \sqrt{n}$ users transmitting, r users receiving
- gains from transmitters to each node: $g_{ik}, \quad i = 1, \ldots, r \quad k = 1, \ldots, n$
- gains from each node to receivers: $h_{ki}, \quad k = 1, \ldots, n \quad i = 1, \ldots, r$
- **Assumptions:**
  - $\{g_{ik}, h_{ki}\}$ subject to independent fading. Averaged over fading and point placement, unit-variance, finite fourth-order moment
  - system synchronous
  - each node k knows its local connections, g_{ik} and h_{kj}, but not the rest of the network

Technical Assumption



Fix four points 1, 2, 3, 4 in A and consider a variable point 5. Then averaged over the fading and the point placement of 5 we assume:

$$Eg_{15}g_{25}g_{35}g_{45} = 0.$$

This is true if the g_{ij} are zero-mean when averaged over the fading:

$$Eg_{15}g_{25}g_{35}g_{45} = E_5\left[E_{|5}g_{15}g_{25}g_{35}g_{45}\right] = E_5\,\underbrace{E_{|5}g_{15}}_{0}\,\underbrace{E_{|5}g_{25}}_{0}\,\underbrace{E_{|5}g_{35}}_{0}\,\underbrace{E_{|5}g_{45}}_{0} = 0.$$

Back to "Listen/Transmit" Protocol

We now have interference. Let's look at our previous protocol.

1. The listen interval: Each of the r transmit users transmits the signal s_i with power p. All other nodes are silent and measure the signals

$$q_k = \sum_{i=1}^{r} s_i g_{ik} + v_k, \quad i = 1, \ldots, r \quad k = 1, \ldots, n$$

2. The transmit interval: Each of the nodes now has to transmit a signal. Let us assume that all it can transmit is a scaled version of what it has previously received:

$$t_k = d_k q_k, \quad k = 1, \ldots, n$$

such that the transmit power is σ_s^2. But how to choose d_k?

Defining the transmit vector $s = \begin{bmatrix} s_1 & s_2 & \dots & s_r \end{bmatrix}$, the receive vector $y = \begin{bmatrix} y_1 & y_2 & \dots & y_r \end{bmatrix}$ and the matrices

$$G = \begin{bmatrix} g_{11} & g_{12} & \dots & g_{1n} \\ g_{21} & g_{22} & \dots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{r1} & g_{r2} & \dots & g_{rn} \end{bmatrix} \quad , \quad H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1r} \\ h_{21} & h_{22} & \dots & h_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \dots & h_{nr} \end{bmatrix}$$

we may write

$$y = s \cdot G \cdot \mathrm{diag}(d_1, \dots, d_n) \cdot H$$

Note that if $n \geq r^2$, the d_i can be chosen such that

$$G \cdot \mathrm{diag}(d_1, \dots, d_n) \cdot H = I_r, \tag{1}$$

which means that the channel is diagonalized and the interference suppressed!

But this can only be done if each node knows all the network gains so that it can solve (1). This is not allowed.

What to do?

- Let each node estimate each of the r transmitted signals:

$$\hat{s}_i = g_{ik}^* q_k, \quad i = 1, \ldots, r \quad k = 1, \ldots, n$$

  Of course, this will be a lousy estimate since the SINR (signal-to-interference-ratio) is $\frac{p}{\sigma_n^2 + (r-1)p}$.

- Let each node attempt to "coherently add" these estimates:

$$t_k = \frac{1}{\sqrt{r}} \cdot \frac{\sigma_s}{\sqrt{\sigma_n^2 + rp}} \sum_{i=1}^{r} h_{ki}^* \hat{s}_i = \frac{\sigma_s}{\sqrt{r(\sigma_n^2 + rp)}} \sum_{i=1}^{r} h_{ki}^* g_{ik}^* q_k$$

  Note therefore that

$$d_k = \frac{\sigma_s}{\sqrt{r(\sigma_n^2 + rp)}} \sum_{i=1}^{r} h_{ki}^* g_{ik}^*$$

  depends only on *local* knowledge of the network gains.

The received signal at the j-th receiver is:

$$y_j = \sum_{k=1}^{n} t_k h_{kj} + w_j.$$

Close inspection of the received signal reveals that it consists of $n+1$ noise terms (one from each of the n nodes) and nr^2 signal terms, of which

- there are n terms of the desired signal s_j that add up *coherently* (one from each of the n nodes)
- there are $n(r-1)$ terms of the desired signal s_j that add up *non-coherently* ($r-1$ from each of the n nodes)
- there are $nr(r-1)$ interference terms (r from each interferer and each node)

Therefore

$$\mathrm{SINR} = \frac{\frac{(n^2+n(r-1))\sigma_s^2 p}{r(\sigma_n^2+rp)}}{\frac{nr(r-1)\sigma_s^2 p}{r(\sigma_n^2+rp)} + \frac{n\sigma_n^2\sigma_s^2}{r(\sigma_n^2+rp)} + \sigma_n^2} \approx \frac{rpn^2\sigma_r^2}{r^2(\sigma_n^2+rp)(\sigma_n^2+n\sigma_r^2)}$$

Precise Statement: Ad-Hoc Network

Assuming the technical condition:

Theorem 3 *Consider a fixed area A and randomly select n nodes and $r \leq O(n^{\frac{1-\epsilon}{2}})$ transmit/receive pairs in this area. Then*

$$\mathit{Prob}\left(\eta > k_1\sqrt{n}\right) > 1 - \frac{k_2}{n^\epsilon},$$

where k_1 and k_2 are positive constants independent of n, but that depend on A and the fading statistics.

Without the technical condition:

Theorem 4 *Consider a fixed area A and randomly select n nodes and $r \leq O(n^{\frac{1-\epsilon}{3}})$ transmit/receive pairs in this area. Then*

$$\mathit{Prob}\left(\eta > k_1\sqrt{n}\right) > 1 - \frac{k_2}{n^\epsilon},$$

where k_1 and k_2 are positive constants independent of n, but that depend on A and the fading statistics.

Power Efficiency of Ad-Hoc Networks

- *provided* $r \leq O(\sqrt{n})$, the power efficiency is $O(\sqrt{n})$
- if we fix the rate for each of the r transmit/receive pairs, then the total power consumption of the network reduces by a factor of $\frac{1}{\sqrt{n}}$.
- this is as good as a sensory network
- rather than diminishing rewards, we are reaping benefits from the increase in the size of the network
- there is built-in fairness: the r transmitters transmit with power $p = O(\frac{1}{\sqrt{n}})$ and all other nodes with power $\sigma_s^2 = O(\frac{r}{n} \cdot \frac{1}{\sqrt{n}})$
- again, the protocol is double-hop and exploits, rather than avoids, interference

Summary and Final Remarks

For cellular networks

- proposed a simple scheme requiring minimal feedback that allows linear increase in capacity in number of transmit antennas
- optimal number of transmit antennas appears to be $\log n$
- exploits multi-user diversity, but also has built-in fairness

For sensory and ad-hoc networks

- if one focuses on power efficiency it apparently does pay off to consider sensory and wireless ad-hoc networks
- the net benefit is larger than the sum of the individual benefits
- to get encouraging asymptotics, rather than attempting to see how the rate scales, the key is to fix the rate and study how the power can be lowered
- interference is, in fact, useful