EE160 Handout #6
January 28, 2014 Due February 4, 2014

Homework Set #3

1. In class we showed that if n_I and n_Q are independent zero-mean Gauusian random variables with variance σ^2, then the joint distribution of the envelope and phase of

$$A\cos(2\pi f_c t) + n_I\cos(2\pi f_c t) - n_Q\sin(2\pi f_c t) = r\cos(2\pi f_c t + \psi),$$

is given by

$$p_{R,\Psi}(r,\psi) = \frac{r}{2\pi\sigma^2} e^{-\frac{r^2+A^2-2Ar\cos\psi}{2\sigma^2}}.$$

We further computed the marginal distribution of the envelope, r, and showed that it had a Rician density. In this problem we are interested in the marginal density of the phase ψ.

(a) Show that

$$p_\Psi(\psi) = \frac{e^{-\frac{A^2}{2\sigma^2}}}{2\pi} + \frac{A}{\sigma}\cdot\frac{\cos\psi}{\sqrt{8\pi}}\cdot e^{-\frac{A^2}{2\sigma^2}\sin^2\psi}\mathrm{erfc}(-\frac{A}{\sigma}\cos\psi),$$

where $\mathrm{erfc}(\cdot)$ is the *complementary error function* defined as

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du.$$

(b) Using the function `erfc(·)` in MATLAB plot the marginal density for the values $\frac{A}{\sigma} = 0.01, 0.1, 1$ on a single graph, as well as for the values $\frac{A}{\sigma} = 1, 5, 20$ on another single graph. Briefly comment on the results.

2. Show that

$$\int_0^\infty \frac{r^3}{\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) dr = A^2 + 2\sigma^2.$$

(Hint: Do not attempt to directly compute the integral. Try to find a physical interpretation.)