Homework Set #1

1. Determine whether the following random processes are *strictly* stationary:

   (a) A process of independent random variables (i.e, a process for which the random variables $X(t_1), \ldots, X(t_k)$ are independent for any k and any $t_1, \ldots t_k$).

   (b) A process of independent indentically-distributed (iid) random variables.

   (c) A process of *exchangeable* random variables. (A set of random variables $X_1, \ldots X_k$ is called exchangeable if their joint probability density function is invariant to any permutation of its arguments, i.e.,

   $$p_{X_1,\ldots,X_k}(x_1, \ldots, x_k) = p_{X_1,\ldots,X_k}(x_{i_1}, \ldots, x_{i_k})$$

   where $(i_1, \ldots, i_k)$ is any permutation of $(1, \ldots, k)$. A process of exchangeable random variables is one for which the random variables $X(t_1), \ldots, X(t_k)$ are exchangeable for any k and any $t_1, \ldots t_k$.)

   (d) The process

   $$Y(t) = X(t)\cos(\omega t + \theta),$$

   where $X(t)$ is a strictly stationary process and θ is a random phase uniformly distributed between 0 and 2π. (*Recall in class we showed that if $X(t)$ is wide-sense stationary, then so is $Y(t)$.*)

   (e) The process

   $$Y(t) = \cos X(t),$$

   where $X(t)$ is a strictly stationary process.

2. Problem 1.1 in Haykin.

3. Problem 1.6 in Haykin.